Journal of Control & Instrumentation
Volume 1, Issue 1-3 Compilation 2011

STM JOURNALS

A CON: EVO Reduction Technique for Uncertain Interval Systems Using Differential Evolution Algorithm

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ABSTRACT

In this paper advantage of evolutionary technique is used in combination with a reliable conventional technique to reduce the dimension of high order uncertain linear interval systems into low order interval systems. The proposed technique is depicted by a name CON: EVO, which stands for combination of conventional & evolutionary technique. In evolutionary method recently proposed Differential Evolution (DE) optimization technique is used with and without powerful conventional Routh approximation technique to minimize the integral square error (ISE) between transient responses of original higher order model and reduced order model pertaining to a unit step input. The algorithm is simple, rugged and computer oriented. It is shown that the algorithm has several advantages, e.g. the reduced order models retain the steady-state value and stability of the original system. The proposed method is illustrated through a numerical example and the results of proposed algorithms are compared with already presented Genetic algorithm method and available $\delta - \gamma$ *Routh approximation conventional technique for interval system proposed by Bandyopadhyay.*

Keywords: *Uncertain interval systems, Differential evolution, Reduced order modeling, Integral square error.*

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INTRODUCTION

Many physical processes in science and engineering are modeled accurately using finite dimensional dynamical systems. However, the discretization in space leads to a high-dimensional system of ordinary differential equations and its transient and harmonic analysis takes too much time even with modern hardware. One scheme to ameliorate this is model order reduction. Model reduction seeks to replace a large-scale system of differential or difference equations by a system of substantially lower dimensions that has nearly the same response characteristics. Reduction of high order systems to lower order models has been an important subject area in control engineering for many years.

From last two decades, much effort has been made in the field of model reduction of fixed coefficients linear dynamic systems and several methods like: Aggregation method [1], Pade approximation [2], Routh approximation [3], Moment matching technique [4], Routh stability technique [5], and L^{∞} optimization technique [6], have been proposed. Among them Routh stability technique has been recognized as the simplest and powerful method because of its ability to yield stable reduced models for stable high-order systems. Further numerous methods of order reduction are also available in the literature [7-14], which are based on minimization of the ISE criterion.Control of real industrial processes is almost always burden with an uncertainty, most practical systems, such as flight vehicles, electric motors, and robots are formulated in

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continuous-time uncertain settings. The uncertainties in these systems arise from unmodelled dynamics, parameter variation, sensor noises, actuator constraints, influences of some external conditions etc. These variations do not follow any of the known probability distribution in general, and are most often quantified in terms of amplitude and/or frequency bounds. The basic case of an uncertainty structure is the interval uncertainty. The essential condition is the independence of its structure. Hence, practical systems or plants are most suitably represented by parametric or uncertain interval models [15–16], instead of deterministic mathematical models. B. Bandyopadhyay has proposed two conventional techniques to reduced order of interval systems, one is Routh-Pade approximation for interval system [23], and another $\gamma - \delta$ Routh approximation for interval systems [22]. Both method suffer from stability point of view because of recursive nature of interval arithmetic used to construct Routh table, which modified by Dolgin [24] to some extent.

In recent years Evolutionary techniques have been attracted much attention from the researchers due to its utilizing analogies with nature or social systems. Differential Evolution grew out of Ken Price's [17], attempts to solve the Chebychev Polynomial fitting Problem that had been posed to him by Rainer Storn in 1995. DE is a very simple population based stochastic function minimizer for global optimization capable of handling nondifferentiable, nonlinear and multi-modal objective function with few, easily chosen, control parameters. As DE can capable of handling nondifferentiable function, so it does not require the gradient of the problem being optimized, as is required by classic optimization methods such as gradient descent and quasi- newton methods. DE can therefore also be used on optimization problems that are not even continuous, are noisy, change over time, etc. It has demonstrated its usefulness and robustness in a variety of application such as Neural network learning, Filter design and the optimization of aerodynamics shapes. DE optimizes a problem by maintaining a population of candidate solutions and creating new candidate solutions by combining existing ones according to its simple formulae, and then keeping whichever candidate solution has the best score or fitness on the optimization problem at hand. DE differs from other Evolutionary Algorithms (EA) in the mutation and recombination phases. DE uses weighted differences between solution vectors to change the population whereas in other stochastic techniques such as Genetic Algorithm (GA) and Expert Systems (ES), perturbation occurs in accordance with a random quantity. DE employs a greedy selection process with inherent elitist features. Also it has a minimum number of EA control parameters, which can be tuned effectively [17, 18].

In this paper, DE is used in two ways to reduce the order of linear interval systems. First way is the mixed method, where DE combines with a CFE conventional technique [20], in this proposed method denominator of reduced interval system determined by the Routh approximation through Alpha table and numerator coefficients are determined by using DE. Second way employs only DE, where order reduction made by minimizing Integral square error (ISE) between transient parts of original and reduced interval systems. Finally the results of two proposed method are compared with already presented genetic algorithm based method [19] and available $\delta - \gamma$ Routh approximation technique proposed by B. Bandyopadhyay for order reduction of interval system [22].

PROCEDURE

A. Problem formulation

Given an original interval system of order 'n' that is described by the transfer function G(s) and its reduced interval model R(s) of order 'k' be represented as:

$$G(s) = \frac{[B_0^-, B_0^+] + [B_1^-, B_1^+]s + [B_2^-, B_2^+]s^2 + \cdots + [B_{n-1}^-, B_{n-1}^+]s^{n-1}}{[A_0^-, A_0^+] + [A_1^-, A_1^+]s + [A_2^-, A_2^+]s^2 + \cdots + [A_n^-, A_n^+]s^n} = \frac{N(s)}{D(s)} \quad (1)$$

Where$[B_i^-, B_i^+]$, $i = 0,1,2, \ldots, n-1$ and$[A_i^-, A_i^+]$, $i = 0,1,2, \ldots, n$ are the interval coefficients of higher order numerator and denominator polynomials respectively.

The objective function is to find a reduced k^{th} order reduced model R(s) such that it mimics the important characteristic of G(s) for the same type of inputs.

$$R(s) = \frac{[b_0^-, b_0^+] + [b_1^-, b_1^+]s + [b_2^-, b_2^+]s^2 + \cdots + [b_{k-1}^-, b_{k-1}^+]s^{k-1}}{[a_0^-, a_0^+] + [a_1^-, a_1^+]s + [a_2^-, a_2^+]s^2 + \cdots + [a_k^-, a_k^+]s^k} = \frac{N_k(s)}{D_k(s)}; where\ k = 1,2, \cdots n \quad (2)$$

Where$[b_i^-, b_i^+]$, $i = 0,1,2, \ldots, k-1$ and$[a_i^-, a_i^+]$, $i = 0,1,2, \ldots, k$ are the interval coefficients of lower order numerator and denominator polynomials respectively.

The rules of the interval arithmetic for two interval coefficients $[a, b], [c, d]$, have been defined as follows [21].

Addition:

$$[a, b] + [c, d] = [a + c, b + d]$$

Subtraction:

$$[a, b] - [c, d] = [a - d, b - c]$$

Multiplication:

$$[a, b] * [c, d] = [Min(ac, ad, bc, bd), Max(ac, ad, bc, bd)]$$

Division:

$$\frac{[a,b]}{[c,d]} = [a, b]\left[\frac{1}{d}, \frac{1}{c}\right], provide\ 0 \neq [c, d] \quad (3)$$

B. Determination of R(s) by mixed evolutionary technique

The reduction procedure by mixed method with DE employing CFE conventional technique described in the following steps

Step-1 Determination of reduced denominator

$$D(s) = [A_0^-, A_0^+] + [A_1^-, A_1^+]s + [A_2^-, A_2^+]s^2 + \cdots + [A_n^-, A_n^+]s^n$$

Let the order of D(s) be even, then using the approximation method [3, 23], D(s) can be expanded into continued fraction expansion (CFE).

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$$D(s) = \cfrac{1}{1+[\alpha_1^-,\alpha_1^+]\frac{1}{s}+\cfrac{1}{[\alpha_2^-,\alpha_2^+]\frac{1}{s}+\ddots+\cfrac{1}{[\alpha_n^-,\alpha_n^+]\frac{1}{s}}}} \quad (4)$$

Eq. (4) is then truncated after first k quotients to produce.

$$D_k(s) = \cfrac{1}{1+[\alpha_1^-,\alpha_1^+]\frac{1}{s}+\cfrac{1}{[\alpha_2^-,\alpha_2^+]\frac{1}{s}+\ddots+\cfrac{1}{[\alpha_k^-,\alpha_k^+]\frac{1}{s}}}} \quad (5)$$

A reduced denominator $D_k(s)$ is then formulated by the denominator of the rational function obtained from inverting the continued fraction (5). The parameter $[\alpha_i^-,\alpha_i^+]$ $(i = 1,2,3 \ldots n)$ are the ratios of consecutive entries in the first column of the Alpha Routh table. These are related with $D_k(s)$ by a set of recursive relations [20].

$$D_1(s) = 1 + [\alpha_1^-,\alpha_1^+]s$$

$$D_2(s) = 1 + [\alpha_2^-,\alpha_2^+]s + [\alpha_1^-,\alpha_1^+][\alpha_2^-,\alpha_2^+]s^2$$

$$D_3(s) = 1 + ([\alpha_1^-,\alpha_1^+] + [\alpha_3^-,\alpha_3^+])s + [\alpha_2^-,\alpha_2^+][\alpha_3^-,\alpha_3^+]s^2 + [\alpha_1^-,\alpha_1^+][\alpha_2^-,\alpha_2^+][\alpha_3^-,\alpha_3^+]s^3$$

$\vdots$

$$D_k(s) = [\alpha_k^-,\alpha_k^+]sD_{k-1}(s) + D_{k-2}(s)\,, \quad k = 1,2,3,\ldots \quad (6)$$

And

$$D_{-1}(s) = D_0(s) = 1$$

Where $[\alpha_k^-,\alpha_k^+]$ parameters are obtained using Alpha Routh Table [3]

Step-2 Determination of reduced numerator

Reduced numerator polynomial determined by the minimizing integral square error between original system $G(s)$ and reduced system $R(s)$ by using DE. Where the denominator polynomial is already obtained by CFE using Eq. (6)

The deviation of the reduced order system from the original system response is given by the error index 'ISE' known as integral square error, which is given as follow:

$$ISE = \int_0^\infty [g(t) - r(t)]^2 dt \quad (7)$$

Where g(t) and r(t) are the unit step response of the original and reduced order interval systems, respectively.

C. Work Procedure of DE for minimization

For the purpose of minimizing of Eq. (7), routine from DE optimization toolbox are used. DE is a stochastic, population-based optimization algorithm introduced by Storn and Price in 1996 [17]. DE works with two populations; old generation and new generation of the same population. The size of the population is adjusted by the parameter N_p. The population consists of real valued vectors with dimension D that equals the number of design parameters/control variables. The population is randomly initialized within the initial parameter bounds. The optimization process is conducted by means of three main operations: mutation, crossover and selection. In each generation, individuals of the current population become target vectors. For each target vector, the mutation operation produces a mutant vector, by adding the weighted difference between two randomly chosen vectors to a third vector.

The crossover operation generates a new vector, called trial vector, by mixing the parameters of the mutant vector with those of the target vector. If the trial vector obtains a better fitness value than the target vector, then the trial vector replaces the target vector in the next generation. The evolutionary operators are described in [17, 18]. The computational flowchart of the differential evolution algorithm is shown in fig. 1

D. Determination of R(s) Using DE

DE can be used alone to produced reduced order interval system without employing any conventional technique. Reduction procedure takes place by minimizing function (7).

III NUMERICAL EXAMPLE

A numerical example is chosen from the literature for the comparison of the lower order system with the original high order system.

Consider a 7th order system taken from literature [25]:

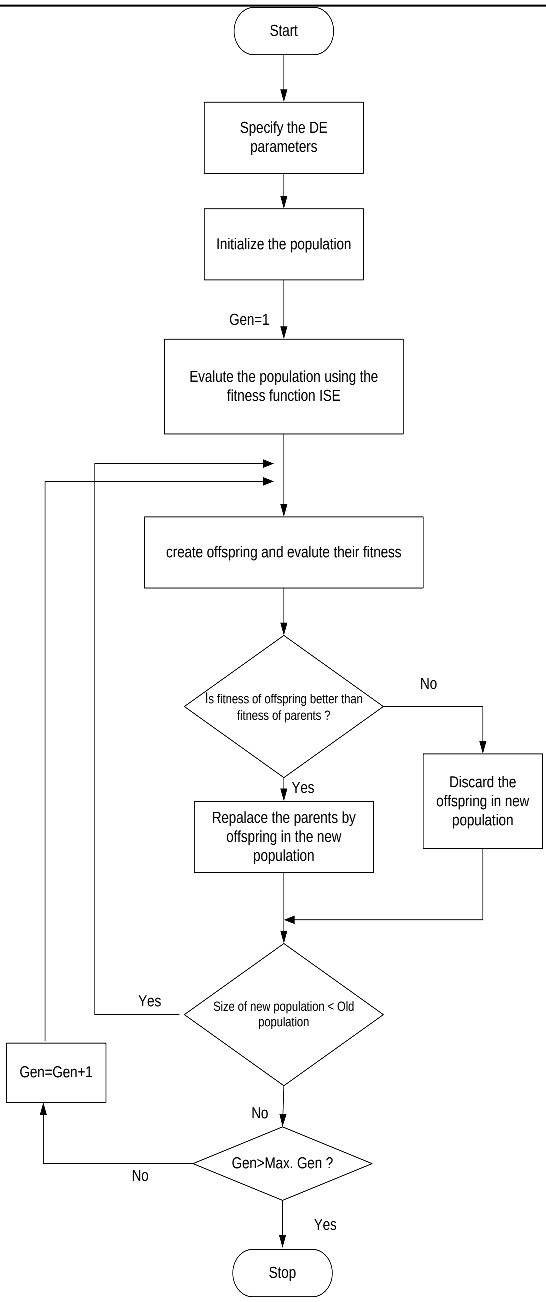

Fig. 1 computational flowchart of the Differential Evolution algorithm

$[\alpha_1^-, \alpha_1^+] = [.159, .19]$ $[\alpha_2^-, \alpha_2^+] = [.576, .749]$ $[\alpha_3^-, \alpha_3^+] = [1.279, 1.989]$ $\vdots$	$[57.352, 63.389]$ $[325.28, 359.52]$ $[480.06, 564.51]$ $[283.84, 375.16]$ $\vdots$	$[572.47, 632.73]$ $[429.02, 474.18]$ $[171.90, 193.82]$ $\cdots$	$[182.875, 202.125]$ $[52.231, 57.729]$ $\cdots$	$[8.779, 9.703]$ $[.95, 1.05]$

$N(s) = [1.9, 2.1]s^6 + [24.7, 27.3]s^5 + [157.7, 174.3]s^4 + [541.975, 599.025]s^3 + [929.955, 1027.845]s^2 + [721.81, 797.79]s + [187.055, 206.745]$

And

$D(s) = [.95, 1.05]s^7 + [8.779, 9.703]s^6 + [52.231, 57.729]s^5 + [182.875, 202.125]s^4 + [429.02, 474.18]s^3 + [572.47, 632.73]s^2 + [325.28, 359.52]s + [57.352, 63.389]$

Objective is to determine the 2nd order reduced interval model which imitate important characteristics of G(s)

a. Determination of R(s) from mixed method

Step-1 – Denominator of reduced order interval system determined by CFE, creating Alpha Routh-table for denominator of G(s) from Table-I.

Necessity of obtain the complete Alpha table diminish with this method, i.e. to find 2nd order model only two values of alpha,$[\alpha_1^-, \alpha_1^+]$, $[\alpha_2^-, \alpha_2^+]$ are needed.

This property makes this method simple in terms of computational efforts.

From Eq. (6)

$D_2(s) = 1 + [\alpha_2^-, \alpha_2^+]s + [\alpha_1^-, \alpha_1^+][\alpha_2^-, \alpha_2^+]s^2$

$= 1 + [.576, .749]s + [.091, .142]s^2$

Step-2 Determination of reduced numerator polynomial from DE

$N_2(s) = [.01, .0930]s + [3.05, 3.075]$

Thus $R_{2M}(s) = \frac{N_2(s)}{D_2(s)} =$

$$\frac{[.01, .0930]s + [3.05, 3.075]}{1 + [.576, .749]s + [.091, .142]s^2}$$

b. Determination of R(s) from Evolutionary approach

To determine the R(s) DE run its routine and the typical parameter for DE optimization routines, used in present study are given.

Table-II- DE Parameters used in present study

Name	Value(type)
Number of generation	100
Population size	80
DE step size	.8
Crossover probability constant	.7
Strategy	experiments

$$R_{2D}(s) = \frac{[9.34, 9.936]s + [4.656, 5.023]}{[1.741, 1.875]s^2 + [5.533, 5.913]s + [1.421, 1.534]}$$

RESULTS

a. Comparison of Reduced models

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Comparison of Reduced interval model $R_2(s)$		ISE for lower limit	ISE for lower limit
Proposed mixed method $R_{2M}(s)$	$\frac{[.01,.0930]s+[3.05,3.075]}{1+[.576,.749]s+[.091,.142]s^2}$	1.6033	1.3685
Proposed Evolutionary approach $R_{2D}(s)$	$\frac{[9.34,9.936]s+[4.656,5.023]}{[1.741,1.875]s^2+[5.533,5.913]s+[1.421,1.534]}$	.0971	.0969
B. Bandyopadhyay reduced[22] $R_{2B}(s)$	$\frac{[1.16,1.84]s+[.27,.53]}{s^2+[.52,.83]s+[.08,.16]}$	2.3295	.9523
GA reduced[19] $R_{2G}(s)$	$\frac{[364.7,429.7]s+[271.7,293.2]}{[61.5,68.99]s^2+[255.7,374.1]s+[83.8,87.67]}$	.1198	.7250

b. Simulation Results

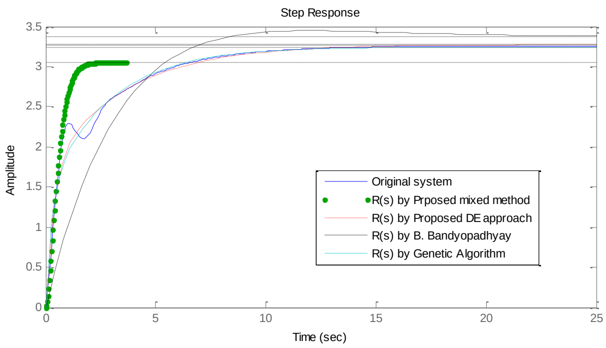

Fig. 2a-Comparison of Step response for lower limit

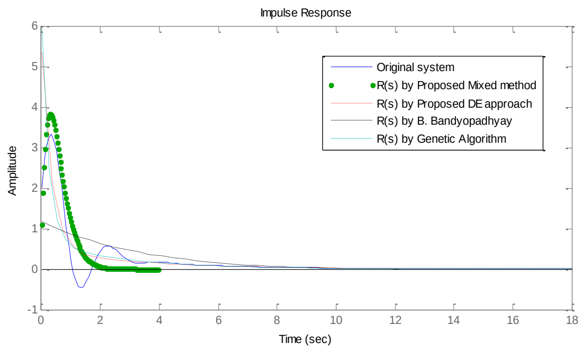

Fig. 3a-Comparison of Impulse response for lower limit

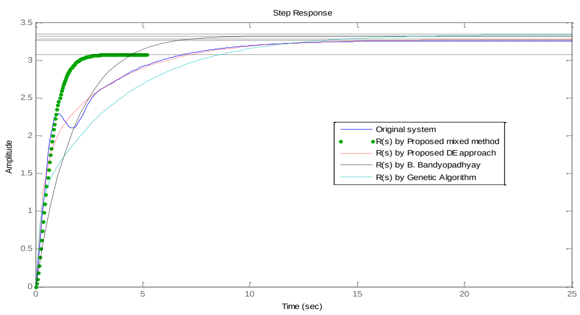

Fig. 2b-Comparison of Step Response for Upper Limit.

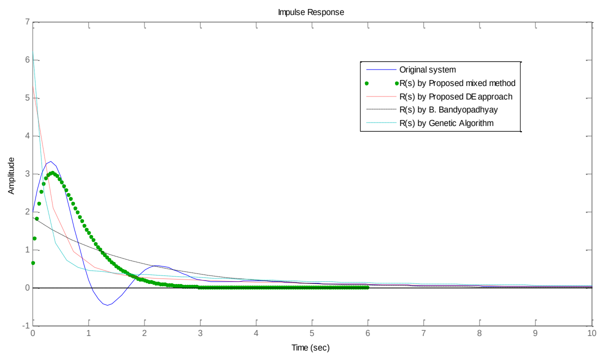

Fig. 3b-Comparison of Impulse response for Upper Limit.

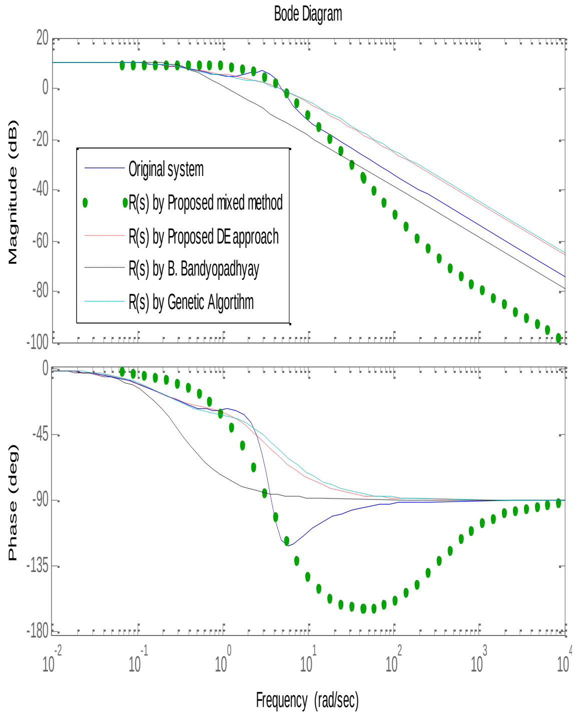

Fig. 4a- Comparison of Bode Response for Lower Limit.

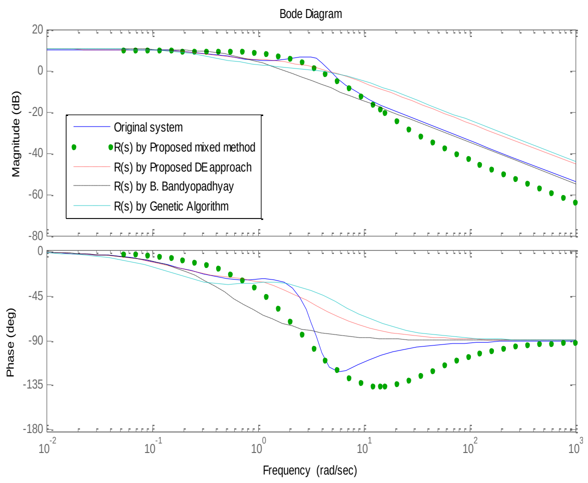

Fig. 4b- Comparison of Bode Response for Upper Limit.

CONCLUSION

In this paper, authors proposed two approaches for order reduction of linear interval systems. This paper combines the advantageous features of evolutionary technique to optimize the function with wide options and in used reliable conventional technique continued fraction expansion (CFE). Order reduction using CFE minimizes the necessity to determine the complete Alpha table with regards to order of reduced model obtained, this feature makes this technique simple in terms of computational efforts.

In evolutionary technique recently proposed Differential Evolution (DE) optimization technique is employed. DE method is based on the minimization of Integral square error (ISE) between the transient response of original high order Interval system and reduced order interval model pertaining to unit step input. Proposed approaches illustrated through a numerical example and a comparison of proposed approaches with recently published GA method and available 'Routh approximation for interval systems' has been presented. From the comparison table and simulation it is observed that DE provide very good approximation of higher order interval system and a reduced order model can be obtain from CFE with very less computational efforts.

The stability of reduced model by proposed approaches has been checked by four kharitonov polynomials [26], and both model found to be robust stable.

APPENDIX

Table-I- Alpha Routh table

$[\propto_1^-, \propto_1^+] = \frac{[A_{00}^-, A_{00}^+]}{[A_{10}^-, A_{10}^+]}$	$[A_{00}^-, A_{00}^+] = [A_0^-, A_0^+]$	$[A_{01}^-, A_{01}^+] = [A_2^-, A_2^+]$	$[A_{02}^-, A_{02}^+] = [A_4^-, A_4^+]$	...
	$[A_{10}^-, A_{10}^+] = [A_1^-, A_1^+]$	$[A_{11}^-, A_{11}^+] = [A_3^-, A_3^+]$	$[A_{12}^-, A_{12}^+] = [A_5^-, A_5^+]$	...
$[\propto_2^-, \propto_2^+] = \frac{[A_{10}^-, A_{10}^+]}{[A_{20}^-, A_{20}^+]}$	$[A_{20}^-, A_{20}^+] = [A_{01}^-, A_{01}^+] - [\propto_1^-, \propto_1^+][A_{11}^-, A_{11}^+]$	$[A_{21}^-, A_{21}^+] = [A_{02}^-, A_{02}^+] - [\propto_1^-, \propto_1^+][A_{12}^-, A_{12}^+]$	$[A_{22}^-, A_{22}^+] = [A_{03}^-, A_{03}^+] - [\propto_1^-, \propto_1^+][A_{13}^-, A_{13}^+]$	...
$[\propto_3^-, \propto_3^+] = \frac{[A_{20}^-, A_{20}^+]}{[A_{30}^-, A_{30}^+]}$	$[A_{30}^-, A_{30}^+] = [A_{11}^-, A_{11}^+] - [\propto_2^-, \propto_2^+][A_{21}^-, A_{21}^+]$	$[A_{31}^-, A_{31}^+] = [A_{11}^-, A_{11}^+] - [\propto_2^-, \propto_2^+][A_{22}^-, A_{22}^+]$	...	
$[\propto_4^-, \propto_4^+] = \frac{[A_{30}^-, A_{30}^+]}{[A_{40}^-, A_{40}^+]}$	$[A_{40}^-, A_{40}^+] = [A_{21}^-, A_{21}^+] - [\propto_3^-, \propto_3^+][A_{31}^-, A_{31}^+]$	$[A_{41}^-, A_{41}^+] = [A_{21}^-, A_{21}^+] - [\propto_3^-, \propto_3^+][A_{32}^-, A_{32}^+]$	...	
$[\propto_5^-, \propto_5^+] = \frac{[A_{40}^-, A_{40}^+]}{[A_{50}^-, A_{50}^+]}$	$[A_{50}^-, A_{50}^+] = [A_{31}^-, A_{31}^+] - [\propto_4^-, \propto_4^+][A_{41}^-, A_{41}^+]$	...		
$\vdots$	$\vdots$			

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