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Wavelet based Adaptive Sliding Mode Control for Discrete Time Uncertain Nonlinear Systems

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ABSTRACT

This paper focuses on the development of a wavelet based adaptive sliding mode control strategy for a class of discrete time uncertain nonlinear systems. An adaptive sliding mode control is utilized to assure the stable tracking of uncertain nonlinear system under consideration. Wavelet neural network (WNN) is used to mimic the uncertainties present in the system. Proposed scheme is derived to guaranty the necessary and sufficient reaching condition for sliding mode control in presence of modeling uncertainties and mathematical inaccuracies. A numerical example is provided to verify the effectiveness of theoretical development.

Keywords: *Adaptive sliding mode control; wavelet networks; discrete time nonlinear systems.*

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INTRODUCTION

Designing of control strategies for discrete time nonlinear systems is an active area of research since last decade. Inclination of researchers towards the designing of discrete time controllers is mainly due to the facts that now a days controller are mainly implemented on DSP chips and digital computers [1–3]. In discrete time systems information about the system are available only at specific time instances and control inputs can only be changed at these time instances. Due to this finite time delay in control computation discrete time controller design is more complicated in comparison to its continuous time counterpart. [1]

Sliding mode control (SMC) is normally used controlling strategy for uncertain towards uncertainties and disturbances along with order reduction. The SMC is variable structure control which drives state trajectories toward a specific hyperplane and maintains the trajectories sliding on hyperplane until the origin of the state space is reached [16]. Feasibility of this scheme is the requirement of infinite rate switching control which is not possible in case discrete time systems. This problem caused by discrete-time implementation of SMC algorithms, has been addressed by several researchers. Few effective strategies for the development of SMC algorithms for discrete-time systems have been sited in the literature [3, 4]. The effectiveness of these schemes lies in the fact that they preserve the distinguished features of sliding mode control and at the same time limits the undesired effects of chattering[5, 6].

Unmodelled dynamics of the plants usually degrades the performance of the controller, especially for nonlinear and complex control problems [8]. In case of systems having perturbed or unknown system dynamics the conventional control strategies are combined with system identification tools, like neural networks for the effective control of the system. In these control strategies the parameters are adaptively tuned by using some suitable adaptation laws. Neural Networks (NNs) have been proved a very efficient system identification tool due to its universal approximation property and leaning capability.[9] Some researchers have developed Wavelet Neural Networks which are having superior approximation capabilities than conventional neural networks due to their space and frequency localization properties. A wavelet network consists of single layer of translated and dilated versions of mother wavelet function. Hence these networks can be considered as optimal approximators [10–12]. Recently the researchers are inclined towards the designing aspects of Wavelet based adaptive controllers for continuous time nonlinear systems [13–15] and their discrete time counterparts. [7]

This paper deals with the designing of adaptive sliding mode controller for a class of discrete time uncertain nonlinear systems. The controller strategy is proposed with an objective to provide an efficient solution to tracking problem of uncertain discrete systems inspired by the approximation capabilities of the wavelet neural networks [13]. This work utilizes the WNN as system identification tool.

The paper is organized as follows: section II highlights the approximation features of WNN, system formulation is described in section III and controller designing and stability aspects are discussed in section IV. Effectiveness of the proposed strategy is illustrated through an example in section V while section VI concludes the paper.

FUNDAMENTALS OF WAVELET NETWORKS

Wavelet Neural Network

Wavelet networks have emerged as a promising tool in the field of learning based control methodology due to its properties like multiresolution and orthonormality. Wavelet network is a single layer network consisting of translated and dilated versions of orthonormal father and mother wavelet function. Basis functions are used in wavelet network span $L^2(\Re)$ subspace. Due to its universal approximation property any function $f(x) \in L^2(\Re)$ can be approximated by linear combination of basis functions. [10, 11] Orthonormality of wavelet bases assures that coefficient needed for reconstruction of any function are fixed and unique and can be tuned independent of other wavelet bases.

Wavelets are derived from the basic requirement of multiresolution analysis, which provides a mathematical framework to describe the increment in information from coarse approximation to finer approximation. Multiresolution analysis is basically a decomposition of space $S \in L^2(\Re)$, with following properties [12]

(a) Whole space S is constructed as a sequence of nested and closed finite dimensional subspace S_i

$$\cdots \subset S_{-1} \subset S_0 \subset S_1 \cdots \subset S_2 \subset \cdots \qquad \forall n \in Z$$

(b) $\bigcap_{n \in Z} S_n = \{0\}$

(c) $\bigcup_{n \in Z} S_n = L^2(\Re)$

So any function $f \in S$ can be approximated with desired accuracy by its projection $f_i = P_i f$ on S_i, i.e., $\lim_{i \to \infty} f_i = f$.

(d) $\begin{array}{l} f(x) \in S_i \Leftrightarrow f(2x) \in S_{i+1} \\ f(x) \in S_i \Leftrightarrow f(x - 2^{-i}k) \in S_i \end{array}$

(e) Multiscale structure provides an orthogonal split of S_{i+1} into low and high frequency parts S_i and W_i respectively.

$$S_{i+1} = S_i \oplus W_i$$

$$W_i \perp W_j \quad \text{if } i \neq j$$

$$W_i \subset S_j \quad \text{if } j > i$$

Decomposition of the whole space S can be expressed as

$$S = S_i \oplus W_i \oplus W_{i-1} \oplus W_{i-2} \cdots \oplus W_0 \oplus W_{-1} \cdots$$

Normally, the wavelet bases are derived using dyadic translation and binary dilation of scaling function $\phi \in S$ and wavelet function $\varphi \in S$. At any resolution j

$$\phi_{jq}(x) = 2^{j/2}\phi(2^j x - q) \qquad j, q \in Z$$

$$S_j = span\{\phi_{jq}(x), q \in Z\}$$

and $\varphi_{jq}(x) = 2^{j/2}\varphi(2^j x - q) \qquad j, q \in Z$

$$W_j = span\{\varphi_{jq}(x), q \in Z\}$$

It follows that any function $f(x(k))$ in S can be expressed as a wavelet series expansion

$$f(x(k)) = \sum_{j=N1}^{N2} \sum_{q=M1}^{M2} \langle \phi_{j,q}(x(k)), f(x(k)) \rangle \phi_{j,q}(x(k)) \qquad (1)$$

Convergence of the wavelet series can be expressed as

$$\lim_{\substack{N1,M1 \to -\infty \\ N2,M2 \to +\infty}} \left\| f(x(k)) - \sum_{j=N1}^{N2} \sum_{k=M1}^{M2} \langle \varphi_{j,q}(x(k)), f(x(k)) \rangle \varphi_{j,q}(x(k)) \right\| = 0 \qquad (2)$$

For nonlinear system modeling the structure of the wavelet network can not be taken infinitely large so truncating the wavelet series to finite numbers of resolutions and translates at each resolution the above expression can be approximated as

$$f(x(k)) = \sum_{j \geq J}^{N} \sum_{q=M1_j}^{M2_j} \langle \phi_{j,q}(x(k)), f(x(k)) \rangle \phi_{j,q}(x(k)) + \varepsilon(x(k)) \qquad (3)$$

where J is lowest resolution, $N \in \square$ represents the highest resolution while $q = [M1_j, \cdots, M2_j] \in \square$ represents the number of translates at jth resolution and $\varepsilon(x(k))$ is the approximation error defined as

$$\varepsilon(x(k)) = f(x(k)) - \sum_{j \ge J}^{N} \sum_{q=M1_j}^{M2_j} \left\langle \varphi_{j,q}(x(k)), f(x(k)) \right\rangle \varphi_{j,q}(x(k)) \tag{4}$$

$$f((x(k))) = \begin{Bmatrix} \sum_{q=M1_J}^{M2_J} \left\langle \phi_{J,q}(x(k)), f(x(k)) \right\rangle \phi_{J,q}(x(k)) + \\ \sum_{j \ge J}^{N} \sum_{q=M1_j}^{M2_j} \left\langle \varphi_{j,q}(x(k)), f(x(k)) \right\rangle \varphi_{j,q}(x(k)) + \varepsilon(x(k)) \end{Bmatrix} \tag{5}$$

Owing to the property of multi resolution analysis (3) can be expressed as

For a function of the form $f(x(k)) : \Re^n \to \Re$, wavelet network model can be extended to multidimensional wavelet network by tensor product of single dimensional wavelet bases. [13]

$$\phi_{J,q}(x(k)) = \prod_{i=1}^{n} \phi_{J,q}(x_i(k)); \quad \varphi_{j,q}(x(k)) = \prod_{i=1}^{n} \varphi_{j,q}(x_i(k)) \tag{6}$$

$$f(x(k)) = \begin{Bmatrix} \sum_{q=M1_J}^{M2_J} \left\langle \phi_{J,q}(x(k)), f(x(k)) \right\rangle \phi_{J,q}(x) + \\ \sum_{j \ge J}^{N} \sum_{q=M1_j}^{M2_j} \left\langle \varphi_{j,q}(x(k)), f(x(k)) \right\rangle \varphi_{j,q}(x(k)) + \varepsilon(x(k)) \end{Bmatrix}$$

$$= \begin{Bmatrix} \sum_{q=M1_J}^{M2_J} \alpha_{J,q}(k) \phi_{J,q}(x(k)) + \sum_{j \ge J}^{N} \sum_{q=M1_j}^{M2_j} \beta_{j,q}(k) \varphi_{j,q}(x(k)) \\ + \varepsilon(x(k)) \end{Bmatrix} \tag{7}$$

where $\alpha_{J,q}(k)$ $\beta_{j,q}(k)$ are weights of wavelet basis functions

Now (7) can be rewritten as

$$f(x(k)) = \alpha^T(k)\phi(x(k)) + \beta^T(k)\varphi(x(k)) + \varepsilon(x(k)) \tag{8}$$

where

$\alpha(k) = \left[\alpha_{JM1_J}(k), .., \alpha_{JM2_J}(k) \right]^T$ and

$\beta(k) = \left[\beta_{JM1_J}(k), \cdots, \beta_{JM2_J}(k), \cdots, \beta_{NM1_N}(k), \cdots \beta_{NM2_N}(k) \right]^T$ are the scaling and wavelet weight vectors respectively.

$$\varphi(x(k)) = \begin{bmatrix} \varphi_{JM1_J}(x(k)), \cdots, \varphi_{JM2_J}(x(k)), \cdots \\ \cdots, \varphi_{NM1_N}(x(k)), \cdots, \varphi_{NM2_N}(x(k)) \end{bmatrix}^T$$

and $\phi(x(k)) = \left[\phi_{JM1_J}(x(k)), .., \phi_{JM2_J}(x(k)) \right]^T$ are wavelet and scaling vectors respectively.

It can be shown that, for an arbitrary constant $\lambda > 0$, there exist a finite integer J_N and real constant optimal weight vectors α^*, β^* such that the unknown nonlinear function $f(x(k))$ can be approximated as follows

$$f(x(k)) = \alpha^{*T}\phi(x(k)) + \beta^{*T}\varphi(x(k)) + \varepsilon(x(k)) \ \forall x(k) \in \Omega \subset \Re^n \tag{9}$$

where $\varepsilon(x(k))$ denotes the approximation error and is assumed to be bounded by $|\varepsilon(x(k))| \le \varepsilon^*$, in which ε^* is a positive constant and Ω is a compact set.

Optimal parameter vectors needed for best approximation of the function are difficult to determine so defining an estimate function as

$$\hat{f}(x(k)) = \hat{\alpha}^T \phi(x(k)) + \hat{\beta}^T \varphi(x(k)) \tag{10}$$

where $\hat{\alpha}, \hat{\beta}$ are the estimates of α^*, β^* respectively. Defining the estimation error as

$$\tilde{f}(x(k)) = f(x(k)) - \hat{f}(x(k)) = \left\{ \tilde{\alpha}^T(k)\phi(x(k)) + \tilde{\beta}^T(k)\varphi(x(k)) + \varepsilon(x(k)) \right\} \tag{11}$$

where $\tilde{\alpha}(k)=\alpha^{*}-\hat{\alpha}(k), \tilde{\beta}(k)=\beta^{*}-\hat{\beta}(k)$

By properly selecting the number of resolutions, the estimation error $\tilde{f}(x(k))$ can be made arbitrarily small on the compact set so that the bound $\left\|\tilde{f}(x(k))\right\| \le \tilde{f}_m$ holds for all $x \in \Omega \subset \Re^n$.

The residual part $\varepsilon(x)$ can be assumed to be bounded by a linear in parameter function

$$|\varepsilon(x(k))| \le \gamma^T z(k) \qquad (12)$$

where $\gamma \in \Re^4$ represents unknown optimal weight vector while $z(k)$ is defined as $z(k)=\left[1, \left\|x(k)\right\|, \left\|x(k)\right\| \left\|\hat{\alpha}(k)\right\|, \left\|x(k)\right\| \left\|\hat{\beta}(k)\right\|\right]^T$.

Assuming that $\hat{\gamma}(k)$ be the estimate of γ, estimation error will be $\tilde{\gamma}(k)=\gamma-\hat{\gamma}(k)$ .Adaptation laws for the online tuning of $\hat{\alpha}(k), \hat{\beta}(k)$ and $\hat{\gamma}(k)$ will be derived in following section.

SYSTEM FORMULATION

Consider a discrete time nonlinear system of the form

$$\begin{aligned}
x_1(k+1) &= x_2(k)+f_1(x(k)) \\
x_2(k+1) &= x_3(k)+f_2(x(k)) \\
&\vdots \\
x_n(k+1) &= f_n(x(k))+u(k) \\
y(k) &= x_1(k)
\end{aligned}$$

(13)

where $x(k)=\left[x_1(k), x_2(k), ..., x_n(k)\right]^T \in \Re^n, u(k) \in \Re, y(k) \in \Re$ are state vector, control input and output respectively. Vector field $f(x(k))=[f_1(x(k)), f_2(x(k)), \cdots, f_n(x(k))]: \Re^n \to \Re^n$ is the unknown nonlinear system dynamics. In this work unknown system dynamics is approximated by a wavelet network.

The objective is to design adaptive sliding mode controller to achieve the desired tracking performance simultaneously nullifying the effect of modeling inaccuracies.

WAVELET SLIDING MODE CONTROLLER DESIGN

Let $\bar{y}_d(k) \in \Re^n$ be the desired trajectory vector and assuming that its past values for previous $(n-1)$ instances are known.

Defining the state tracking error vector

$$e(k)=x(k)-\bar{y}_d(k) \qquad (14)$$

with $e_i(k)=x_i(k)-y_d(k+i-n) \qquad i=1,\cdots,n$

So the error dynamics of the system (13) becomes

$$\begin{aligned}
e_i(k+1) &= e_{i+1}(k)+f_i(x(k)) \quad 1 \le i \le n-1 \\
e_n(k+1) &= f(x(k))+u(k)-y_d(k+1)
\end{aligned}$$

(15)

A linear functional sliding surface is defined as

$$s(k)=ce(k)$$

(16)

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where $c=\left[c_1,c_2,\cdots,c_n\right]\in\Re^n$ is a vector of positive constant values, selected such that the poles of the systems are located inside the unit circle. Then $s(k+1)$ is defined as

$$s(k+1)=\left\{\begin{matrix} c_1e_2(k)+c_2e_3(k)+\cdots+c_{n-1}e_n(k)+c_1f_1(x(k)) \\ +c_2f_2(x(k))+\cdots+c_nf_n(x(k))+c_n(u(k)-y_d(k+1)) \end{matrix}\right\} \quad (17)$$

In this expression the component $(c_1f_1(x(k))+\cdots+c_nf_n(x(k)))$ is modeled by using a wavelet network.

For discrete time systems an inequality of the form

$$s(k)\left[s(k+1)-s(k)\right]<0 \quad (18)$$

is necessary but not sufficient condition to be used as reaching law, as it does not assures the convergence towards the sliding surface.

In order to assure reaching condition constraint imposed by following inequality is also required to be satisfied

$$\left|s(k+1)\right|<\left|s(k)\right| \quad (19)$$

By combining (18) and (19) an efficient sliding mode control law can be constructed. [6]

Defining the control effort as

$$u(k)=u_{eq}(k)+u_r(k) \quad (20)$$

where the equivalent control tern is defined as

$$u_{eq}(k)=\begin{pmatrix} y_d(k+1)-\frac{1}{c_n}(c_1e_2(k)+c_2e_3(k)+\cdots+c_{n-1}e_n(k)- \\ \hat{f}(x(k))-\mu s(k)) \end{pmatrix} \quad (21)$$

here $\hat{f}(x(k))$ is the wavelet approximation of uncertain term $(c_1f_1(x(k))+c_2f_2(x(k))+\cdots+c_nf_n(x(k)))$ and $0<\mu<1$.

The robust control term is defined as

$$u_r(k)=\frac{1}{c_n}(-\hat{\gamma}^T(k)z(k)\operatorname{sgn}(s(k))) \quad (22)$$

With the help of the proposed tuning laws presented in the next part of this section, the error term $\tilde{f}(k)$ is reduced to a small arbitrary value which is further attenuated by robust control term $u_r(k)$.

Weight update rules for wavelet network parameters and weight parameters for adaptive approximation of residual term are based on Lyapunov based adaptation methodology and are given as

$$\hat{\alpha}(k+1)=\hat{\alpha}(k)+\Delta\hat{\alpha}(k)$$
$$\hat{\beta}(k+1)=\hat{\beta}(k)+\Delta\hat{\beta}(k)$$
$$\hat{\gamma}(k+1)=\hat{\gamma}(k)+\Delta\hat{\gamma}(k)$$

$$\begin{aligned}\Delta\alpha(k)&=-\tau_1s(k)\phi(x(k))\\ \Delta\beta(k)&=-\tau_2s(k)\varphi(x(k))\\ \Delta\gamma(k)&=-\tau_3\left|s(k)\right|z(k)\end{aligned} \quad (23)$$

where τ_1,τ_2,τ_3 are the learning rates with positive constants.

Theorem: For the system of the form (13), with sliding surface (16) , if weight parameters are adaptively tuned as per laws proposed in (23) then the wavelet based sliding mode control law (20), (21) and (22) guarantees the convergence of every trajectory of closed loop system to the sliding surface satisfying the inequalities (18) and (19).

Proof: Consider a function of the form

$$\Delta V(k)=s(k)(s(k+1)-s(k))+\frac{1}{\tau_1}\tilde{\alpha}^T(k)(\tilde{\alpha}(k+1)-\tilde{\alpha}(k))+\frac{1}{\tau_2}\tilde{\beta}^T(k)(\tilde{\beta}(k+1)-\tilde{\beta}(k))+\frac{1}{\tau_3}\tilde{\gamma}^T(k)(\tilde{\lambda}(k+1)-\tilde{\gamma}(k)) \quad (24)$$

Substituting control law $u(k)$ (20), (21) in above equation

$$\Delta V(k)=s(k)(\tilde{f}-\mu s(k)+c_n u_r)+\frac{1}{\tau_1}\tilde{\alpha}^T(k)(\hat{\alpha}(k+1)-\hat{\alpha}(k))+\frac{1}{\tau_2}\tilde{\beta}^T(k)(\hat{\beta}(k+1)-\hat{\beta}(k))+\frac{1}{\tau_3}\tilde{\gamma}^T(k)(\hat{\gamma}(k+1)-\hat{\gamma}(k))$$

$$\Delta V(k)=s(k)(\tilde{f}(x(k))-\mu s(k)+c_n u_r)+\frac{1}{\tau_1}\tilde{\alpha}^T(k)\Delta\hat{\alpha}(k)+\frac{1}{\tau_2}\tilde{\beta}^T(k)\Delta\hat{\beta}(k)+\frac{1}{\tau_3}\tilde{\gamma}^T(k)\Delta\hat{\gamma}(k)$$

Substituting $\tilde{f}(x(k))$ (11) and adaptation laws for $\Delta\hat{\alpha}(k)$ and $\Delta\hat{\beta}(k)$ (23) in above equation,

$$\Delta V(k)=s(k)(\varepsilon(x(k))\text{ -}\mu s(k)+c_n u_r)+\frac{1}{\tau_3}\tilde{\gamma}^T(k)\Delta\hat{\gamma}(k)$$

$$\leq |s(k)||\varepsilon(x(k))|\text{-}\mu s^2(k)+c_n u_r s(k)+\frac{1}{\tau_3}\tilde{\gamma}^T(k)\Delta\hat{\gamma}(k)$$

Substituting $|\varepsilon(x(k))|$ (12) in above equation

$$\leq |s(k)|\gamma^T z(k)\text{-}\mu s^2(k)+c_n u_r s(k)+\frac{1}{\tau_3}\tilde{\gamma}^T(k)\Delta\hat{\gamma}(k)$$

$$\leq |s(k)|(\tilde{\gamma}^T(k)+\hat{\gamma}^T(k))z(k)\text{-}\mu s^2(k)+c_n u_r s(k)+\frac{1}{\tau_3}\tilde{\gamma}^T(k)\Delta\hat{\gamma}(k)$$

Substituting u_r and adaptation laws for $\Delta\hat{\gamma}(k)$ (23) in above equation

$$\leq \text{-}\mu s^2(k)$$

(25)

Therefore $\Delta V(k)$ is negative which implies the convergence of system trajectories to sliding surface and boundedness of all the closed loop signals.

SIMULATION RESULTS

Simulation is performed to verify the effectiveness of proposed wavelet based sliding mode control strategy. Considering a system of the form

$$\begin{aligned}
x_1(k+1)&=x_2(k)+\frac{0.3x_1^{\,2}(k)x_2(k)}{5+x_1^{\,2}(k)+x^2{}_3(k)}\\
x_2(k+1)&=x_3(k)+\frac{0.2x_1(k)x_2(k)\sin(x_2(k))}{5+x_2^{\,2}(k)+x^2{}_3(k)} \qquad (26)\\
x_3(k+1)&=\frac{0.62x_1(k)\sin(2x_1(k))}{10+x_2^2(k)}+u(k)\\
y(k)&=x_1(k)
\end{aligned}$$

System belongs to the class of discrete time uncertain nonlinear systems defined by (9) with $n=3$. The sampling time T is taken as 0.05 sec. The proposed controller strategy is applied to this system with an objective to solve the tracking problem of system.

The desired trajectory is taken as

$$y_d=\begin{cases}0.8\,\mathrm{sgn}(\sin 1.3\pi kT) & 0\leq k\leq 400\\ \sin(.5\pi kT) & 400<k\leq 500\\ 2 & k=501\\ -3 & k=502\\ 0.7\sin(.5\pi kT) & 502<k\leq 600\\ 0.7\,\mathrm{sgn}(\sin 2\pi kT) & 600<k\leq 800\\ 0.8\cos(1.5\pi kT) & 800<k\leq 1000\end{cases} \qquad (27)$$

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Initial conditions are taken as $x(0)=[1.8, 1.2, 1.5]^T$. Controller parameters are taken as $c=[0.1, 0.075, 0.234]; \mu=0.1$. Wavelet network used for modeling the uncertainties is constructed by using three dimensional Daubechies wavelet (db3), J is kept 2 with $M_{2_J}=7$ while N is selected as 5 and translates are made double when resolution is increased by 1. Wavelet parameters for wavelet network are tuned online using the proposed adaptation laws, initial conditions for all the wavelet parameters are set to zero. To avoid chattering $\mathrm{sgn}(s(k))$ is replaced by following saturation function

$$r(k)=\begin{cases} s(k) & |s(k)|\le 0.05 \\ \mathrm{sgn}(s(k)) & |s(k)|>0.05 \end{cases} \qquad (28)$$

Simulation results are shown in Figure.1 and Figure.2. Figure.1 reflects the efficient tracking performance of the proposed controller scheme. Due to fast and efficient learning ability of wavelet network, system response rapidly tracks the desired trajectory with rapidly decaying transient observed during initial phase of the simulation. Tracking efficiency of the proposed scheme is also illustrated by inserting bounded spikes in the desired trajectory. Figure.2 shows tracking error and sliding function for the system under consideration. As observed from the figure tracking error and sliding surface are always close to zero with mean square value of the tracking error about $5.4e-3$.

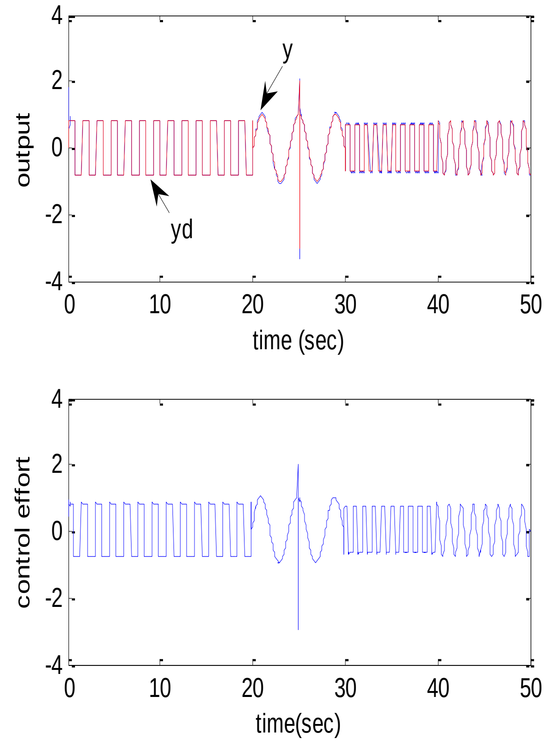

Fig. 1: System Output and Control Effort

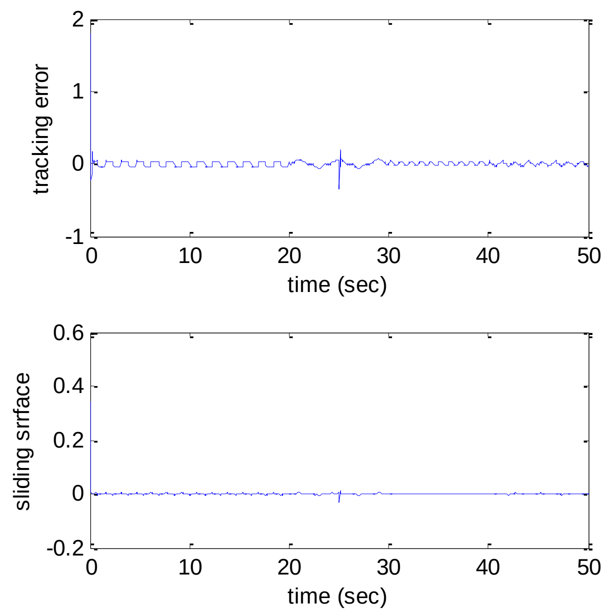

Fig. 2: Tracking Error and Sliding Surface

CONCLUSION

A wavelet based sliding mode control scheme is proposed for a class of discrete time uncertain nonlinear systems. Wavelet networks are used for approximating the uncertain system dynamics. Adaptation laws are developed for online tuning of the wavelet parameters. The theoretical analysis is validated by the simulation results.

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