STM JOURNALS

Journal of Experimental & Applied Mechanics
ISSN: 2230-9845 (Online), ISSN: 2321-516X (Print)
Volume 8, Issue 3
www.stmjournals.com

Isogeometric Topology Optimization of Continuum Structures using Evolutionary Algorithms

N.S.S. Sahithi, K.N.V. Chandrasekhar*, T. Muralidhara Rao
Department of Civil Engineering, CVR College of Engineering, Rangareddy (D), Telangana, India

Abstract

Isogeometric analysis is a popular method for the analysis of problems involving complex geometry and governed by differential equations. Meta-heuristics are widely used to determine the optimum distribution of material within the given design domain. The focus of this study is to perform isogeometric topology optimization of continuum structures using meta-heuristics nature inspired firefly algorithm. NURBS basis functions are used to construct the geometric model and to calculate the displacements as well. In this paper, a two dimensional plate structure is modeled using NURBS basis functions and analyzed for the given loading and boundary conditions. ESO technique is used to identify the elements which carry the material and penalize the remaining elements which do not carry any stress. Few examples have been solved and the results are presented. The results clearly show that the distribution of material using isogeometric analysis is similar to the distribution of material using FEA.

Keywords: *Isogeometric analysis, structural optimization, firefly, metaheuristic, continuum, plate*

****Author for Correspondence*** E-mail: biml.koralla1@gmail.com

INTRODUCTION

Isogeometric analysis has shown advantages over traditional approaches in the context of optimization problems. NURBS can be useful to generate smooth surfaces leading to more physically accurate models [1]. This study is focused on structural mechanics and optimization problems for the purpose of validation and methodology. The first paper on IGA is published by Hughes *et al.* in the year 2005 in which the authors proposed IGA with different refinement techniques [2, 3]. The isogeometric analysis is spreading into various fields and the applications cannot be understood. This study is focused on basic problems in structural mechanics.

Next part of the paper briefly discusses on the literature review in the field of applied mechanics. The following part presents a methodology to conduct this study, and after that the theoretical background to conduct this study is discussed, the flowchart used to write the program in C++ is presented, few problems from the literature are solved to verify the proposed approach and the results are compared with those existing in the literature and the last part presents the conclusions and future study for further analysis and design.

LITERATURE REVIEW

The paper by Hughes in 2005 has revolutionized the field to model the geometry and analyze the domain [2]. Several papers were published in different areas and the isogeometric analysis can be used for an exact and more precise result. Although optimization of civil engineering structures involves lot of programming and computational effort to perform the optimization of continuum structures, there are very few papers in this area of research. Nguyen, in his paper has presented the IGA to solve several problems including static analysis, vibration problems and crack width calculations [1]. Espath in his paper solved non-linear mechanics using IGA [4]. Gondegaon in his paper solved the static analysis and modal analysis using Isogeometric analysis [3]. Gondegaon in another paper applied IGA to solve differential equations such as Poisson’s equation using Galerkin formulation [5]. Hartman in his paper has used IGA with LS-DYNA [6]. Hassani applied IGA to perform topology optimization of continuum structures using optimality criteria [7]. He solved a few problems cantilever carrying a point load at the corner as well as point load at the centre. He has also optimized the problems on simply supported beam carrying a point load at the centre of the lower edge. He optimized the MBB beam problem which is one of the benchmark problems in the optimization of structures. Joo-Sung applied the isogeometric analysis concept in the context of structural mechanics [8]. He optimized a cantilever model having bracket as test problems and the results were compared with the analysis of MSC/Nastran. Shah in his paper has studied the application of IGA to optimize structures in aerospace engineering [9]. He compared the results of his study with the results obtained by using MSC/Nastran. Nagy in his paper applied variational formulation to perform optimization [10].

METHODOLOGY

The number of publications on isogeometric topology optimization of continuum structures is very few. One cannot understand the papers for a complete picture of how to perform the analysis. The theoretical background includes the formulation required and the flowchart shows the steps to perform the optimization process. Figure 1 shows the approach followed to conduct this study.

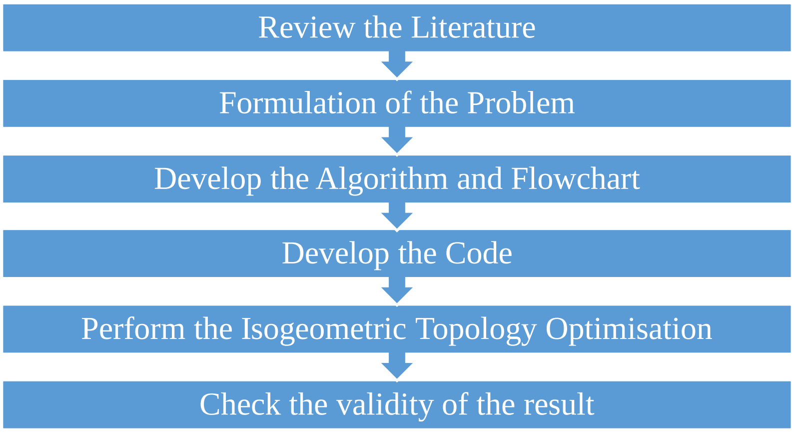


Fig. 1: *Flowchart Showing the Methodology Approach.*

THEORETICAL BACKGROUND

Basis Functions [2]

$$N_{i,0}(\xi) = \begin{Bmatrix} 1 & if\, \xi_i \le \xi < \xi_{i+1} \\ 0 & otherwise \end{Bmatrix}$$

For p=1, 2, 3, …. They are defined by:

$$N_{i,p}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi)$$

This is referred to as the Cox-de Boor recursion formula.

Derivatives of B-Spline Basis Functions

$$\frac{d}{dx} N_{i,p}(\xi) = \frac{p}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) - \frac{p}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi)$$

Generalize to Higher Order Derivatives [7]

$$\frac{d^k}{d^k \xi} N_{i,p}(\xi) = \frac{p}{\xi_{i+p} - \xi_i} \left(\frac{d^{k-1}}{d^{k-1}\xi} N_{i,p-1}(\xi) \right) - \frac{p}{\xi_{i+p+1} - \xi_{i+1}} \left(\frac{d^{k-1}}{d^{k-1}\xi} N_{i+1,p-1}(\xi) \right)$$

B-Spline Curves

$$C(\xi) = \sum_{i=1}^{n} N_{i,p}(\xi) B_i$$

B-Spline Surfaces

$$S(\xi, \eta) = \sum_{i=1}^{n} \sum_{j=1}^{m} N_{i,p}(\xi) M_{j,q}(\eta)\, B_{i,j}$$

B-Spline Solids

$$S(\xi,\eta,\zeta)=\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{l}N_{i,p}(\xi)M_{j,q}(\eta)L_{k,r}(\quad|\quad|\zeta)B_{i,j,k}$$

NURBS Basis Function

With a given projective B-spline curve and its associated projective control points in hand, the control points for the NURBS curve are obtained by using the following relations:

$$(B_i)_j=\frac{(B|\quad|i^w)_j}{w_i}j=1,2\ldots,d$$

$$w_i=(B|\quad|i^w)_{jd+1}$$

NURBS basis is given by:

For NURBS Curve

$$R_i^p(\xi)=\frac{N_{i,p}(\xi)w_i}{\sum_{i=1}^{n}N_{i,p}(\xi)w_i}$$

$$C(\xi)=\sum_{i=1}^{n}R_i^p(\xi)B_i$$

This is identical to the B-Splines.

For NURBS Surfaces

$$R_{i,j}^{p,q}(\xi,\eta)=\frac{N_{i,p}(\xi)M_{j,q}(\eta)w_{i,j}}{\sum_{i=1}^{n}\sum_{j=1}^{m}N_{i,p}(\xi)M_{j,q}(\eta)w_{i,j}}$$

For NURBS Solids

$$R_{i,j,k}^{p,q,r}(\xi,\eta,\zeta)=\frac{N_{i,p}(\xi)M_{j,q}(\eta)L_{k,r}(\zeta)w_{i,j,k}}{\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{l}N_{i,p}(\xi)M_{j,q}(\eta)L_{k,r}(\zeta)w_{i,j,k}}$$

Derivatives of NURBS

Apply the quotient rule,

$$\frac{d}{d\xi}R_i^p(\xi)=w_i\frac{W(\xi)N'_{i,p}(\xi)-W'(\xi)N_{i,p}(\xi)}{\left(W(\xi)\right)^2}$$

$$where N'_{i,p}(\xi)=\frac{d}{d\xi}N_{i,p}(\xi)\wedge W'(\xi)=\sum_{i=1}^{n}N'_{i,p}(\xi)w_i$$

For Higher Order Derivatives of NURBS Basis Functions **[7]**

$$A_i^{(k)}(\xi)=w_i\frac{d^k}{d\xi^k}N_{i,p}(\xi),\left(no\sum on\ i\right)$$

We do not sum on the repeated index, and let,

$$W^{(k)}(\xi)=\frac{d^k}{d\xi^k}W(\xi)$$

Higher order derivatives can be expressed in terms of the lower order derivatives as:

$$\frac{d^k}{d\xi^k}R_i^p(\xi)=\frac{A_i^{(k)}(\xi)-\sum_{j=1}^{k}\binom{k}{j}W^{(j)}(\xi)\frac{d^{(k-j)}}{d\xi^{(k-j)}}R_i^p(\xi)}{W(\xi)}$$

$$where\binom{k}{j}=\frac{k!}{j!\,(k-j)!}$$

Parametric to Parent Mapping

$$\xi = \frac{1}{2}\left[(\xi_{i+1} - \xi_i)\hat{\xi} + (\xi_{i+1} - \xi_i)\right]$$

$$\eta = \frac{1}{2}\left[(\eta_{i+1} - \eta_i)\hat{\eta} + (\eta_{i+1} - \eta_i)\right]$$

$$J_{\hat{\xi},\hat{\eta}} = \frac{1}{4}(\xi_{i+1} - \xi_i)(\eta_{i+1} - \eta_i)$$

***Parametric Space to Physical Space* [3]**

$$X = N_1M_1X_1 + N_2M_1X_2 + N_2M_2X_3 + N_1M_2X_4$$
$$Y = N_1M_1Y_1 + N_2M_1Y_2 + N_2M_2Y_3 + N_1M_2Y_4$$

$$\begin{bmatrix} \frac{\partial N}{\partial \xi} \\ \frac{\partial N}{\partial \eta} \end{bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix} \begin{bmatrix} \frac{\partial N}{\partial x} \\ \frac{\partial N}{\partial y} \end{bmatrix}$$

$$\frac{\partial x}{\partial \xi} = \frac{\partial}{\partial \xi}[N_1M_1N_2M_1N_2M_2N_1M_2][x_1x_2x_3x_4]^T$$

$$\frac{\partial x}{\partial \eta} = \frac{\partial}{\partial \eta}[N_1M_1N_2M_1N_2M_2N_1M_2][x_1x_2x_3x_4]^T$$

$$\frac{\partial y}{\partial \xi} = \frac{\partial}{\partial \xi}[N_1M_1N_2M_1N_2M_2N_1M_2][y_1y_2y_3y_4]^T$$

$$\frac{\partial y}{\partial \eta} = \frac{\partial}{\partial \eta}[N_1M_1N_2M_1N_2M_2N_1M_2][y_1y_2y_3y_4]^T$$

Strain Displacement Matrix [11]

$$B = \begin{bmatrix} \frac{\partial N}{\partial x} & 0 \\ 0 & \frac{\partial N}{\partial y} \\ \frac{\partial N}{\partial y} & \frac{\partial N}{\partial x} \end{bmatrix}$$

$$\epsilon = \begin{bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{bmatrix}$$

$$\begin{bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} \end{bmatrix} = \frac{1}{|J|}\begin{bmatrix} J_{22} & -J_{12} \\ -J_{21} & J_{11} \end{bmatrix} \begin{bmatrix} \frac{\partial u}{\partial \xi} \\ \frac{\partial u}{\partial \eta} \end{bmatrix}$$

$$\epsilon = AG = \frac{1}{|J|}\begin{bmatrix} J_{22} & -J_{12} & 0 & 0 \\ 0 & 0 & -J_{21} & J_{11} \\ -J_{21} & J_{11} & J_{22} & -J_{12} \end{bmatrix}\begin{bmatrix} \frac{\partial u}{\partial \xi} \\ \frac{\partial u}{\partial \eta} \\ \frac{\partial v}{\partial \xi} \\ \frac{\partial v}{\partial \eta} \end{bmatrix}$$

***For Element 1* [3]:**

$$\begin{bmatrix} \frac{\partial u}{\partial \xi} \\ \frac{\partial u}{\partial \eta} \\ \frac{\partial v}{\partial \xi} \\ \frac{\partial v}{\partial \eta} \end{bmatrix} = \begin{bmatrix} 4\eta-2 & 0 & 2-4\eta & 04\eta & 0 & -4\eta & 0 \\ 4\xi-2 & 0 & -4\xi & 04\xi & 0 & 2-4\xi & 0 \\ 0 & 4\eta-2 & 0 & 2-4\eta 0 & 4\eta & 0 & -4\eta \\ 0 & 4\xi-2 & 0 & -4\xi \; 0 & 4\xi & 0 & 2-4\xi \end{bmatrix}\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \\ q_7 \\ q_8 \end{bmatrix}$$

Plane Stress

$$D = \frac{E}{(1-v^2)}\begin{bmatrix} 1 & v & 0 \\ v & 1 & 0 \\ 0 & 0 & (1-v)/2 \end{bmatrix}$$

Plane Strain:

$$D = \frac{E}{(1+v)(1-2v)}\begin{bmatrix} 1-v & v & 0 \\ v & 1-v & 0 \\ 0 & 0 & \left(\frac{1}{2}\right)-v \end{bmatrix}$$

***Stiffness Matrix* [5]**

$$k = t\int_{-1}^{1}\int_{-1}^{1} B^T DB \left|J_{\xi,\eta}\right| d\xi d\eta \left|J_{\acute{\xi},\acute{\eta}}\right| weight$$

Gauss Quadrature

$$\xi = \pm\frac{1}{\sqrt{3}}\eta = \pm\frac{1}{\sqrt{3}}$$

Traction

$$\int u^T T = [uv]^T \begin{bmatrix} T_x \\ T_y \end{bmatrix} \left|J_{\xi,\eta}\right| d\xi d\eta \left|J_{\acute{\xi},\acute{\eta}}\right| weight$$

$$[uv]^T \begin{bmatrix} T_x \\ T_y \end{bmatrix} = \begin{bmatrix} N_1M_1 & 0 \\ 0 & N_1M_1 \\ N_2M_1 & 0 \\ 0 & N_2M_1 \\ N_2M_2 & 0 \\ 0 & N_2M_2 \\ N_1M_2 & 0 \\ 0 & N_1M_2 \end{bmatrix}_{8x2} \begin{bmatrix} N_1M_1 & 0 & N_2M_1 & 0 & N_2M_2 & 0 & N_1M_2 & 0 \\ 0 & N_1M_1 & 0 & N_2M_1 & 0 & N_2M_2 & 0 & N_1M_2 \end{bmatrix}_{2x8} \begin{bmatrix} T_{x1} \\ T_{y1} \\ T_{x2} \\ T_{y2} \\ T_{x3} \\ T_{y3} \\ T_{x4} \\ T_{y4} \end{bmatrix}_{8x1}$$

Algorithm to Perform the IGA Analysis [9]

The algorithm to perform the isogeometric analysis of a two dimensional plate structure carrying in-plane loading:

1. Determine NURBS coordinates using elRangeU and elRangeV.
2. Store the connectivity of the element in an array names sctrB (of size nn).
3. Define strain displacement matrix B of size (1,2*nn).
4. Set k_e=0.
5. Loop over Gauss points (GPs) $\{\xi'_j, \omega'_j\}$ j=1, 2, …, n_{gp} where ngp is the number of gauss points.
   a) Compute parametric coordinate ξ corresponding to ξ'_j.
   b) Compute $|J_{\xi'}|$ corresponding to the equations.
   c) Compute the derivatives of the shape functions $R^e_{w\xi}$ and $R^e_{w\eta}$ at point ξ, η.
   d) Compute J_ξ using control points (sctr(:,e)) $R^e_{w\xi}$ and $R^e_{w\eta}$.
   e) Find J_ξ^{-1} and determinant $|J_\xi|$.
   f) Compute the shape function derivatives $R_x = J_\xi^{-1}[R^T_{,\xi} R^T_{,\eta}]$.
   g) Use Rx to build the strain displacement matrix B.
   h) $k_e = k_e + B^T DB|J_{\xi'}||J_{\eta'}|\omega'_j$.
6. End loop on gauss points.
7. Assemble k_e into global stiffness matrix K^G.
8. End loop over all the elements.

The flowchart to develop the code in C++ is as shown below in Figure 2.

FLOW CHART

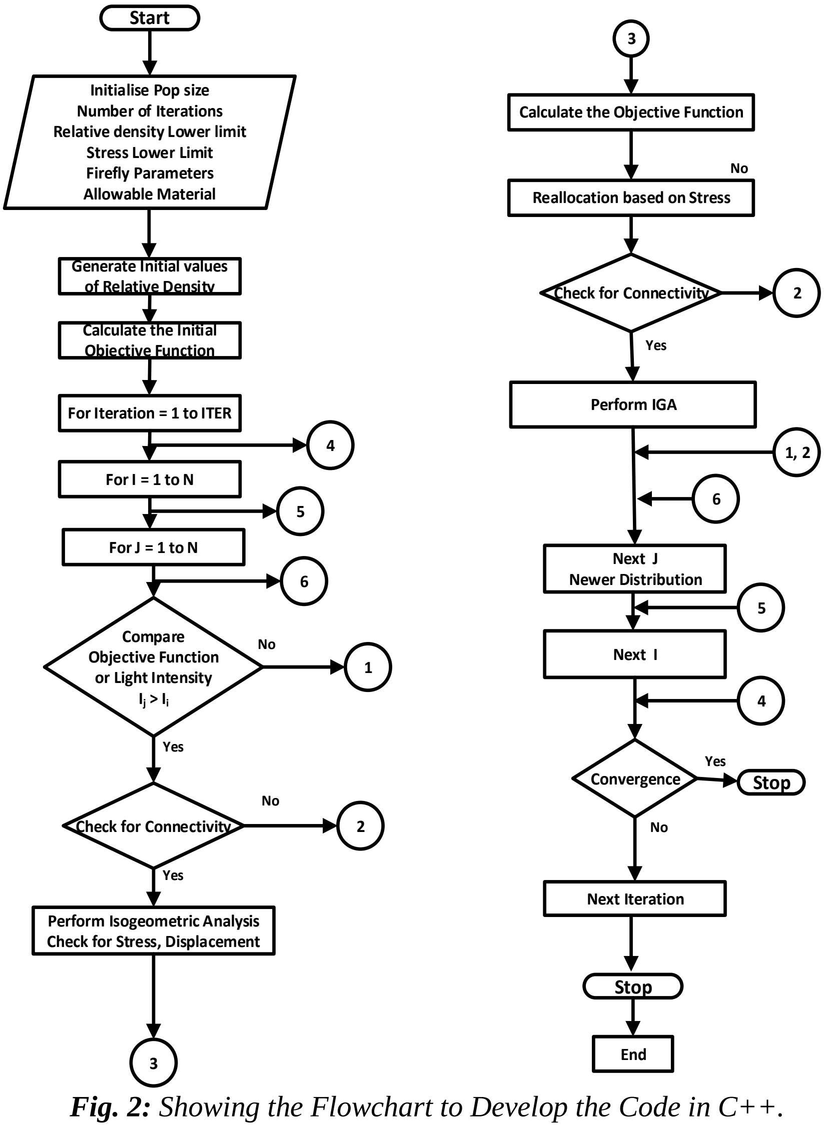
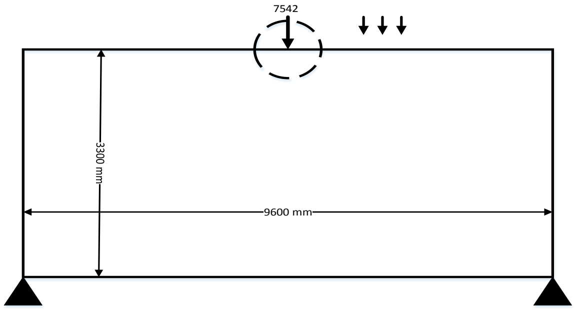
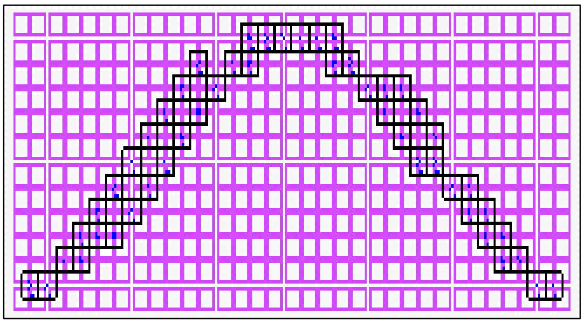


Fig. 2: *Showing the Flowchart to Develop the Code in C++.*





(a) Simply supported Plate *(b) Final distribution of material*

***Fig. 3: (a)** Showing the Simply Supported Plate Carrying in-Plane Loading at the Centre of the Top Edge; **(b)** the Final Optimized Output of Isogeometric Topology Optimization of the Simply Supported Plate Structure Carrying in-Plane Loading.*

ANALYSIS

A Simply Supported Beam

A simply supported plate having dimensions of 9600 mm×3300 mm as shown in the Figure 3a carries a point load of magnitude 7542 N at the centre of the top edge acting vertically downwards. The simple supports are provided at both the ends on the lower edge as shown. The entire domain is meshed using 352 numbers four node quadrilateral elements in plane stress condition. The total number of nodes (control points) is 396. The Young's Modulus of elasticity is taken as 2×10^5 N/mm^2 and the Poisson's ratio as 0.30. The weight density is taken as 7800 kg/m^3. The maximum permissible stress is 200 N/mm^2. The thickness of the plate is one unit. The degree of the basis function along both the dimensions is first order. Figure 3b shows the optimal distribution of material using IGA and firefly algorithm.

The mesh size is 32 elements along dimension 1 and 11 elements along dimension 2. The total number of elements =32×11=352.

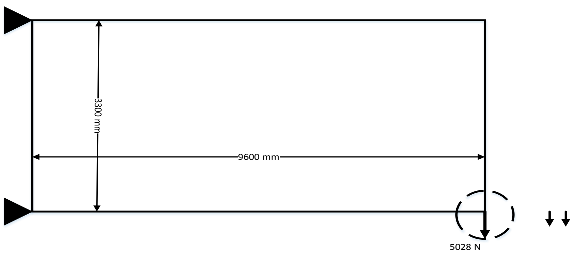
Xi Vector = { 0 0 0
0.031250.0625 0.093750.125
0.156250.1875 0.21875 0.25
0.28125 0.3125
0.34375 0.375 0.40625
0.4375 0.468750.5 0.53125
0.5625 0.59375 0.625
0.65625 0.6875 0.71875
0.75 0.78125 0.8125
0.84375 0.875 0.90625
0.9375 0.968751 1 1
}

Eta Vector = { 0 0 0
0.0909090909 0.1818181818
0.2727272727 0.3636363636
0.4545454545 0.5454545455
0.6363636364 0.7272727273
0.8181818182 0.9090909091 1
1 1 }

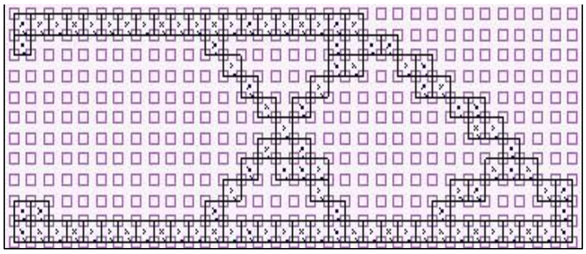
The NURBS basis functions were used and the optimization process was carried out using firefly algorithm as per the flowchart as shown in the Figure 2. The isogeometric topology optimization of the plate structure is performed and the final optimized output is shown in the Figure 3. The total number of iterations is 424.

A Cantilever Plate

A cantilever plate having dimensions 9600 mm×3300 mm is supported at the corners of the left edge as shown in the Figure 4a. The plate carries a point concentrated load of 5028 N at the lower corner of the right end. To avoid stress concentration, the loading and the supports are distributed over two nodes as shown. The thickness of the plate is one unit. The Young's Modulus of elasticity is 2×10 5 N/mm^2. The Poisson's ratio is taken as 0.3. The design domain is discretized using four node first order quadrilateral elements in plane stress condition. The total number of nodes is 396 and the number of elements is 352. The permissible stress is 200 N/mm^2. The weight density of the material is taken as 7800 kg/m^3. The degree of the basis function is linear. Figure 4b shows the optimal distribution of the material in the design domain. Figure 4c shows the optimal distribution of material by Hassani *et al.* using 1000 linear finite elements [7]. Figure 5 shows the iteration curve with weight of the structure on Y-axis and the iteration number on X-axis.



(a) Cantilever Plate



(b) Final Distribution of material

Fig. 4: ***(a)*** *Showing the Initial Design Domain of the Cantilever Plate Carrying a Point Load at the Corner of the Right Edge,* ***(b)*** *Showing the Isogeometric Topology Optimal Distribution of Material Inside the Design Domain of the Cantilever Plate Using Firefly Algorithm.*

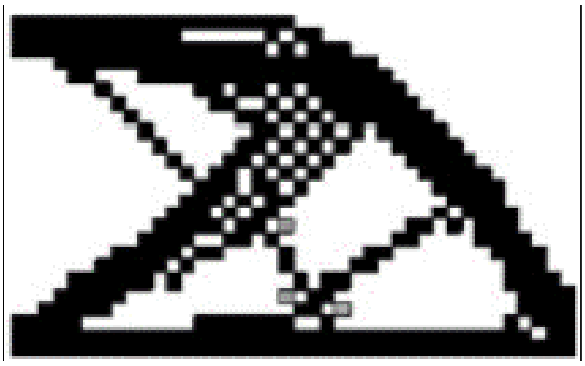


Fig. 4: ***(c)*** *Showing the Optimal Distribution of Short Cantilever Beam with 1000 Four-Node Finite Elements by Hassani et al. [7].*

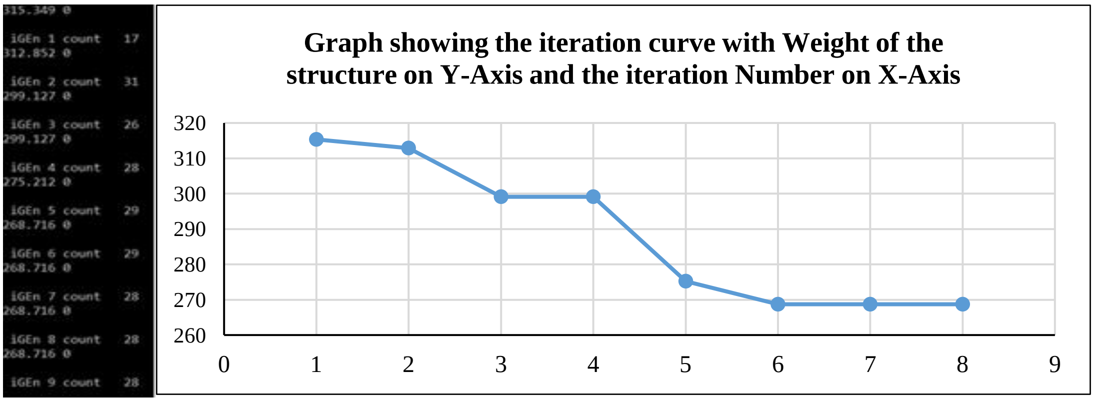


Fig. 5: *Showing the Iteration History and the Iteration Curve with the Weight of the Structure on Y-Axis and the Iteration Number on X-Axis.*

A Cantilever Plate Carrying Point Load at the Mid-Point of the Right Edge

For the sake of the symmetry, one half of the structure is analyzed. A cantilever plate of 9600 mm×3300 mm carries a concentrated load of 5028 N as shown in the Figure 6. The plate is supported at the lower left end corner as shown in the Figure 6. The structure is discretized into 352 elements first order four node plane quadrilateral elements in plane stress condition. The total number of nodes is 396. The thickness of the plate is one unit. The Young's Modulus of elasticity is 2×10^5 N/mm^2 and the Poisson's ratio is 0.3. The permissible stress is taken as 200 MPa and the weight density is 7800 kg/m^3. The degree of basis function is linear. Figure 6b shows the optimal distribution of the material inside the design domain. Figure 7(a) shows the flip vertical image and compared with the existing results obtained as shown in the Figure 7(b) by Hasasni *et al*. using 400 and 1617 control points respectively [7]. Figure 8 shows the iteration history and the iteration curve with weight of the structure on Y-axis and the iteration number on X-axis.

ISSN: 2230-9845 (Online), ISSN: 2321-516X (Print)

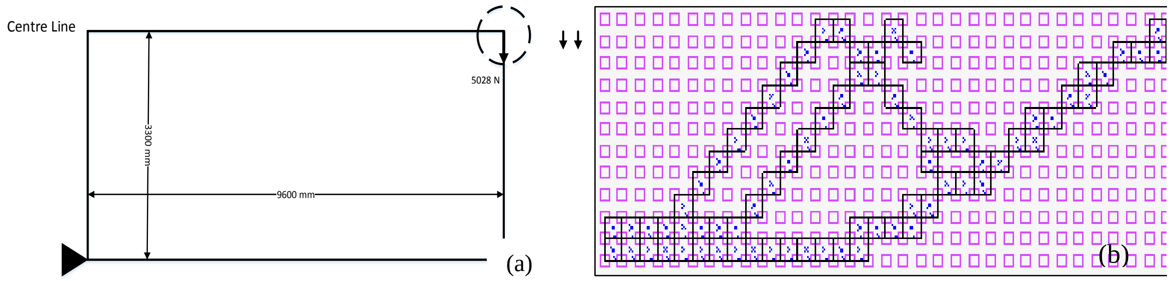
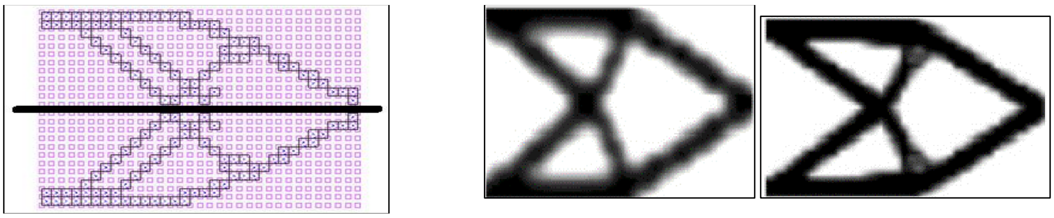


Fig. 6: ***(a)*** *Cantilever Plate Carrying Point Load at the Midpoint,* ***(b)*** *Optimal Distribution of Material.*



(a) (b)

Fig. 7: ***(a)*** *Showing the Flip Vertical Image,* ***(b)*** *Optimal Layout Using 400 Control Points and 1617 Control Points by Hassani et al. [7].*

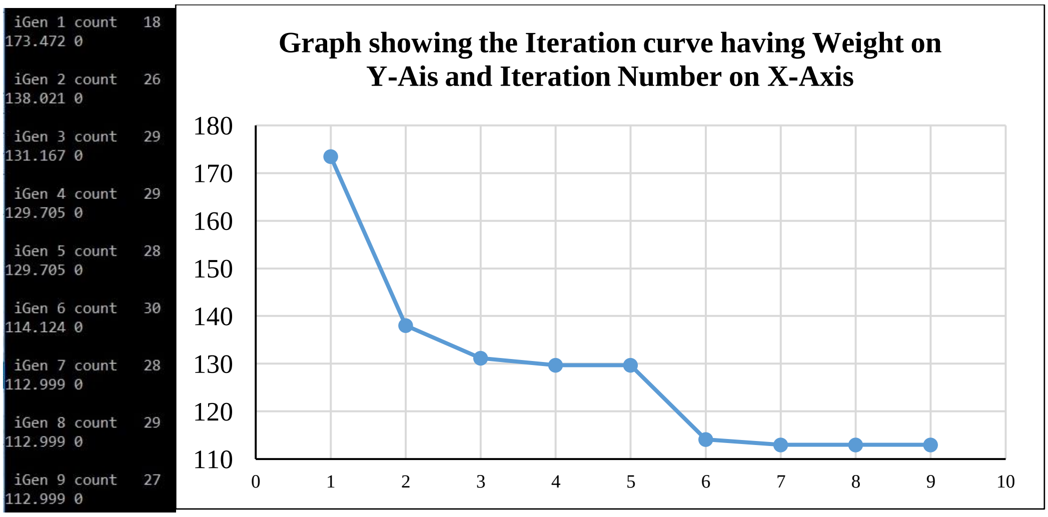


Fig. 8: *Showing the Iteration History and the Iteration Curve with Weight on Y-Axis and the Iteration Number on X-Axis.*

A Simply Supported Beam Carrying a Point Load at the Centre of the Lower Edge between the Supports (Michelle)

The given design domain is a simply supported beam 9600×3300 mm. The beam carries a point load of magnitude 10056 N at the centre. For the sake of simplicity, half of the domain is analyzed due to symmetry. The mesh consists of 352 elements and 396 nodes. The modulus of elasticity is taken as 2×10^5 N/mm^2 and the Poissons ratio as 0.3. The weight density is taken as 7870 kg/m^3. The thickness is one unit. The degree of basis function is one along each direction. The design domain is as shown in the Figure 9(a). The Figure 9(b) shows the optimal distribution of material using the firefly algorithm. The design is a fully stressed design. Figure 10(a) shows the flip horizontal image of the distribution and compared with the optimal distribution as shown in Figure 10(b) by Hartman using 1617 control points (nodes) [6]. Table 1 shows the comparison of the minimum volume obtained using Firefly algorithm and the by Hassani *et al.* [7]. The final weight of the structure is higher, further optimization can reduce the weight which requires additional computational effort.

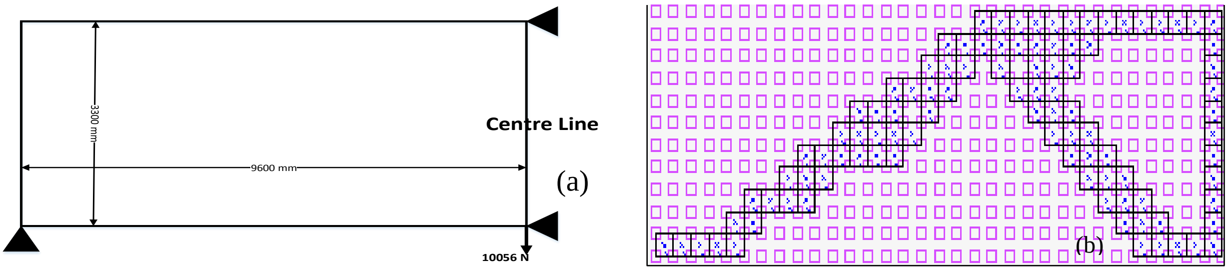


Fig. 9: ***(a)*** *Showing the Design Domain,* ***(b)*** *Optimal Design Using IGA (98/352).*

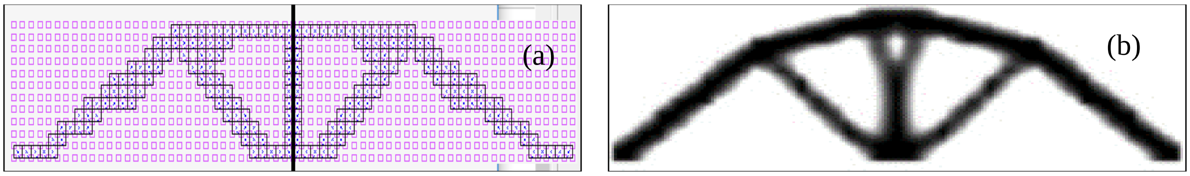


Fig. 10: ***(a)*** *Showing Flip Horizontal Image,* ***(b)*** *Optimal Distribution, 1617 ctrl pts [7].*

Table 1: *Showing the Comparison of the Volume Fraction.*

	This Study using FFA	**Hassani *et al.* [7]**
Minimum Volume (V/V_0) %	27.84%	20%

Limitations

1. Domains with fewer elements are used. However the results show a good agreement in the optimal distribution of material.
2. The analysis is done using single patch only.

CONCLUSIONS

The isogeometric analysis is used to perform the topology optimization of continuum structures. A few basic problems are solved and the results are compared with those existing in the literature. The IGA of the cantilever plate fixed on left end and carries a point load at the right end corner shows a similar distribution with those existing in the literature. The results can be refined using large number of elements. The optimal distribution of a simply supported beam carrying a point load at the centre of the top edge shows that the distribution of the material is an inverted V-form similar to the theoretical distribution. The Michelle beam clearly shows a similar distribution to those existing in the literature. The results using Firefly algorithm show a good agreement with those results in the literature. However single patch is used and domains having fewer elements were used.

Future Study

1. The concept can be applied to solve vibration problems to find the fundamental frequency.
2. IGA can be applied in fracture mechanics to calculate the crack width.
3. IGA can be applied to perform non-linear analysis.

REFERENCES

1. Nguyen, *et al.* Isogeometric Analysis: An Overview and Computer Implementation Aspects. Cornell University, Submitted on 10 May 2012. *Math Comput Simul*, Science Direct, Elsevier. Nov 2015; 117: 89–116p. 10.1016/j.matcom.2015.05.008.
2. Hughes TJR, *et al.* Isogeometric Analysis: CAD, Finite Elements, NURBS, Exact Geometry and Mesh Refinement. *Comput Methods in Appl Mech Eng*. 2005; 194: 4135–4195p.
3. Gondegaon S, Voruganti H. Static Structural and Modal Analysis Using Isogeometric Analysis. *Journal of Theoretical and Applied Mechanics (JTAM)*, Sofia. 2016; 46(4): 36–75p.
4. Espath *et al.* An Introduction to Isogeometric Analysis Applied to Solid Mechanics. *Mecanica Computational*, (Articulo Complete), Argentina. Nov 2011; XXX: 1955–1975p.
5. Gondegaon, Ahmad, Voruganti. Geometric Modelling for Isogoemetric Analysis. *International Conference on Theoretical, Applied and Computational Mechanics, ICTACEM 2014/439*, IIT Kharagpur, India.

6. Hartman *et al.* About Isogeometric Analysis and the New NURBS-Based Finite Elements in LS-DYNA. *8th European LS-DYNA Users Conference*, Strasbourg, France. May 23–24, 2011.
7. Hassani, *et al*. An Isogeometric Approach to Structural Topology Optimization by Optimality Criteria. *Struct Multidisc Optim*. 2012; 45: 223–233p. doi 10.1007/s00158-011-0680-5
8. Joo-Sung Lee, Yoon Nam, Kyoungsik Chang. Optimum Structural Design based on Isogeometric Approach. *IFOST Proceedings*. 2010.
9. Mit Shah, Ravi Katukum. Stress Analysis without Meshing Isogeometric Analysis Finite Element Method. UG Summer Internship, Boeing, India, *International Conference on Innovations in Computer Science and Technology, ICICSIT 2015.*
10. Nagy Attila, *et al*. On the Variation Formulation of Stress Constraints in Isogeometric Design. *Comput Methods in Appl Mech Eng*. 2010; 199: 2687–2696p.
11. Chandrupatla Tirupati R, Belegundu Ashok D. *Introduction to Finite Elements in Engineering*. Pearson Publications; 2011.

Cite this Article
Sahithi NSS, Chandrasekhar KNV, Muralidhara Rao T. Isogeometric Topology Optimization of Continuum Structures using Evolutionary Algorithms. *Journal of Experimental & Applied Mechanics*. 2017; 8(3): 8–18p.