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Stress and Deformation Analysis of Rotating Cylindrical Pressure Vessel of Functionally Graded Material Modeled by Mori-Tanaka Scheme

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Abstract
The present study deals with the linear elastic analysis of rotating cylindrical pressure vessels. The vessels are made up of one directional functionally graded material (FGM), in which mechanical and physical properties are varying along the radial direction. The analysis is carried out using finite element method which is based on the principle of stationary total potential (PSTP). Material properties are graded according to the Mori-Tanaka distribution law and ceramic-metal as well as metal-ceramic both the types of FGMs are considered. The effects of the gradation of material properties on the stress and deformation behavior of the vessels are investigated and a comparison of deformation and stresses for different values of grading index is presented. The results obtained are in good agreement with the established reports and show that there is a significant variation in stresses and deformation behavior of the FGM vessels as compared to homogeneous vessels. Further it is observed that metal-ceramic FGM vessel having n = 0.5 has the lowest overall stresses, and therefore can be most effectively employed for the rotating cylindrical pressure vessels.

Keywords: *Functionally graded material (FGM), linear elastic analysis, annular rotating cylindrical shell, finite element method (FEM)*

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INTRODUCTION
Functionally graded material (FGM) is a multi-phase composite material which has continuous and smooth distribution of physical and mechanical properties along a particular direction. In FGMs material properties are graded by continuously varying the volume fractions of the constituents. Functionally graded structures like rotating disks, rotating pressure vessels, etc. are widely used in nuclear power plants, space vehicles, aircrafts and many other engineering and industrial applications [1]. In rotating pressure vessels the total stress is due to the internal pressure as well as the centrifugal load, both of which acts in radial direction. Thus, control and optimization of total stress and deformation in radial direction is an important task, which can be achieved by using FGMs for pressure vessels.

Many researchers have worked on elastic analysis of rotating conical shells, cylindrical shells, disks, etc. by analytical as well as approximate methods such as finite element method. Naki Tutuncu, Murat Ozturk [2] reported closed form solution for stresses and displacements in functionally graded cylindrical and spherical vessels subjected to internal pressure, using the infinitesimal theory of elasticity. The material stiffness obeying a power law is assumed to vary through the wall thickness and Poisson’s ratio is assumed constant. K. Abrinia, H. Naee, F. Sadeghi and F. Djavanroodi [3] have analyzed FGM thick cylinders under combined pressure and temperature loading. Mohammad Zamani Nejad and Gholam Hosein Rahimi [4] reported work on stresses analysis in isotropic rotating thick-walled cylindrical pressure vessels made of functionally graded materials. The pressure, inner radius and outer radius are considered constant. Material properties are considered as a function of the radius of the cylinder to a power law function and the Poisson’s ratio is assumed as constant. Asemi et al. [5] have applied finite element method to obtain the elastic behavior of functionally graded thick truncated conical shell which is based on Rayleigh–Ritz energy formulation. The study presented by them shows the effects of semi-vertex angle of the cone and the power law exponent on the distribution of displacements and stresses.

In a recent work, S. Ansari Sadrabadi, G.H. Rahimi [6] studied thick-walled cylindrical tanks or tubes made of functionally graded material, under internal pressure and temperature gradient. Material parameters have been considered as power functions. Mehdi Ghannad, Gholam Hosein Rahimi and Mohammad Zamani Nejad [7] worked on elastic analysis of pressurized thick cylindrical shells with variable thickness made of functionally graded materials. Nejad et al. [1, 8, 9] have performed a semi-analytical approach using first-order shear deformation theory (FSDT), Matched asymptotic method (MAM) and multilayer method (MLM), for the purpose of elastic analysis of rotating thick truncated conical shells made of functionally graded materials (FGMs). The cone has finite length, and it is subjected to axisymmetric hydrostatic internal pressure. The inner surface of the cone is pure ceramic and the outer surface is pure metal, and the material composition varying continuously along its thickness.

Present research work reports study on rotating cylindrical pressure vessels subjected to internal pressure. Finite element method based on principle of stationary total potential is adopted for the analysis purpose. The vessels are made up of functionally graded material in which aluminum as a metal and zirconia as a ceramic is taken and gradation of the material properties is obtained by Mori-Tanaka scheme. The work aims at investigating the effects of functionally gradation of the material properties on the deformation and stresses behavior of the vessels for both ceramic-metal and metal-ceramic FGM. At the same time the effects of grading parameter is also find out and presented in the form of graph for some numerical problem.

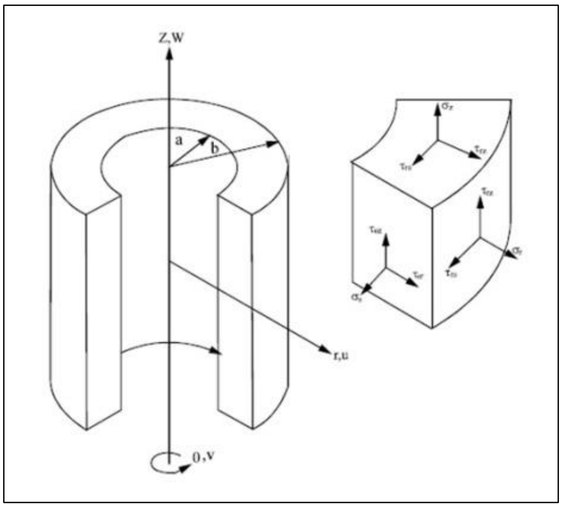

Fig. 1: *Cylindrical Geometry and Stresses on a Cylindrical Element.*

MATERIAL MODELING

The effective bulk modulus and shear modulus of the FGM disk, evaluated using the Mori-Tanaka scheme [10] are given by:

$$B(r) = (B_o - B_i)/V_o\left(1 + (1 - V_o)\frac{3(B_o - B_i)}{3B_i + 4G_i}\right) + B_i \quad (1)$$

$$G(r) = (G_o - G_i)/V_o\left(1 + (1 - V_o)\frac{(G_o - G_i)}{G_i + f_1}\right) + G_i \quad (2)$$

$$f_1 = \frac{G_i(9B_i + 8G_i)}{6(B_i + 2G_i)} \quad (3)$$

Here, V is the volume fraction of the phase material. The subscripts i and o refer to the inner and outer materials respectively. The volume fraction of the inner and outer phases are related by

$$V_i + V_o = 1 \quad (4)$$

And, V_o is expressed as

$$V_o = \left(\frac{r - r_i}{r_o - r_i}\right)^n \quad (5)$$

Where, n $(n \geq 0)$ is the volume fraction exponent. The elastic modulus E can be found as

$$E(r) = \frac{9B(r)G(r)}{3B(r) + G(r)} \quad (6)$$

The mass density ρ can be given by the rule of mixtures as:

$$\rho(r) = (\rho_o - \rho_i)\left(\frac{r - r_i}{r_o - r_i}\right)^n + \rho_i \quad (7)$$

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FINITE ELEMENT MODELING

For axisymmetric problem, an (*r-z*) plane (analogous to *x-y* plane for plane elasticity) can be considered. Figure 1 shows the axisymmetric geometry of cylindrical shell and stresses on a cylindrical element. Four independent nonzero strains exist in axisymmetric problems [11].

$$\varepsilon_r = \frac{\partial u}{\partial r},\ \varepsilon_\theta = \frac{u}{r},\ \varepsilon_z = \frac{\partial v}{\partial z},\ \gamma_{rz} = \frac{\partial u}{\partial z} + \frac{\partial v}{\partial r} \qquad (8)$$

In standard finite element notation, strain displacement relationship can be written as:

$$\{\varepsilon\} = [B]\{\delta\}^e \qquad (9)$$

Where, [*B*] is strain displacement relation matrix which depends on element taken and contains derivatives of shape functions.

From generalized hooks law, components of total strain in radial, circumferential and axial direction is given by

$$\varepsilon_r = \frac{1}{E}\left(\sigma_r - \nu\sigma_\theta - \nu\sigma_z\right) \qquad (10)$$

$$\varepsilon_\theta = \frac{1}{E}\left(\sigma_\theta - \nu\sigma_r - \nu\sigma_z\right) \qquad (11)$$

$$\varepsilon_z = \frac{1}{E}\left(\sigma_z - \nu\sigma_\theta - \nu\sigma_r\right) \qquad (12)$$

By solving above three equations, stress strain relationship can be obtained as follows:

$$\sigma_r = \frac{E(r)}{(1-2\nu)(1+2\nu)}\left[(1-\nu)\varepsilon_r + \nu\varepsilon_z + \nu\varepsilon_\theta\right] \qquad (13)$$

Similarly, axial and circumferential stress can also be obtained.

In standard finite element matrix notation above stress strain relations can be written as:

$$\{\sigma\} = [D(r)]\left(\{\varepsilon\} - \{\varepsilon\}^0\right) + \{\sigma\}^0 \qquad (14)$$

Where,

$$\{\sigma\} = \begin{Bmatrix} \sigma_r \\ \sigma_\theta \\ \sigma_z \\ \tau_{rz} \end{Bmatrix} \qquad (15)$$

$$D(r) = \frac{(1-\nu)E(r)}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1 & \frac{\nu}{(1-\nu)} & \frac{\nu}{(1-\nu)} & 0 \\ \frac{\nu}{(1-\nu)} & 1 & \frac{\nu}{(1-\nu)} & 0 \\ \frac{\nu}{(1-\nu)} & \frac{\nu}{(1-\nu)} & 1 & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2(1-\nu)} \end{bmatrix} \qquad (16)$$

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_r \\ \grave{o}_\theta \\ \varepsilon_z \\ \gamma_{rz} \end{Bmatrix} \qquad (17)$$

Where, *D*(*r*) is a function of radius *r* and $\{\sigma\}^0$ and $\{\varepsilon\}^0$ are initial stress and strain. Taking initial stress and strain zero, equation (14) reduces to

$$\{\sigma\} = [D(r)]\{\varepsilon\} \qquad (18)$$

When the shell rotates and is subjected to internal pressure, it experiences a distributed force over its volume and surface. Under these forces when shell is properly supported (so as to prevent rigid body motion), it undergoes deformation and stores internal strain energy *U*, which is given by:

$$U = \frac{1}{2}\int_v \{\varepsilon\}^T\{\sigma\}\, dv \qquad (19)$$

Also the potential of external body and surface force is given by:

$$V = -\int_v \{\delta\}^T\{q_v\}\,dv - \int_s \{\delta\}^T\{q_s\}\,ds \qquad (20)$$

The element level equation can be written as:

$$U^e = \int_v \frac{1}{2}\{\delta\}^{eT}[B]^T[D(r)][B]\{\delta\}^e\, dv \qquad (21)$$

$$V^e = -\int_v \{\delta\}^{eT}[N]^T\{q_v\}\,dv - \int_s \{\delta\}^{eT}[N]^T\{q_s\}\,ds \qquad (22)$$

The total potential of the element can be written as:

$$\pi_p^e = \int_v \frac{1}{2}\{\delta\}^{eT}[B]^T[D(r)][B]\{\delta\}^e\, dv - \int_v \{\delta\}^{eT}[N]^T\{q_v\}\,dv - \int_s \{\delta\}^{eT}[N]^T\{q_s\}\,ds \qquad (23)$$

Defining element stiffness matrix $[K]^e$ and element load vector $\{f\}^e$ as:

$$[K]^e = \int_v [B]^T[D(r)][B]\,dv \qquad (24)$$

$$\{f\}^e = \int_v \{\delta\}^{eT}[N]^T\{q_v\}\,dv + \int_s \{\delta\}^{eT}[N]^T\{q_s\}\,ds \qquad (25)$$

Total potential energy of the shell is given by:

$$\pi_p = \sum \pi_p^e \qquad (26)$$

Using the principle of stationary total potential (PSTP), the total potential is set to be stationary with respect to small variation in the nodal degree of freedom that is:

$$\frac{\partial \pi_p}{\partial \{\delta\}^T} = 0 \qquad (27)$$

which gives system level equation for shell as:

$$[K]\{\delta\} = \{F\} \qquad (28)$$

Where,

$$[K] = \sum_{n=1}^{N} [K]^e = Globel\ Stiffness\ matrix \qquad (29)$$

$$\{F\} = \sum_{n=1}^{N} [f]^e = Globel\ load\ vector \qquad (30)$$

N = no. of elements.
The summation indicates assembly of individual elemental matrices following the standard procedure of assembly.

RESULTS AND DISCUSSION

Validation

Figure. 2 shows validation of the present work with the results of pre-established reports. To validate the current work, problems of reference [2] are reconsidered and two types of material models are analyzed. Following equations are used for material modeling in reference [2]:

$$V_c = \left(\frac{x-1}{k-1}\right)^n \qquad (31)$$

$$\nu = \nu_c V_c + \nu_m (1 - V_c) \qquad (32)$$

$$E = E_c V_c + E_m (1 - V_c) \qquad (33)$$

Where, V is the volume fraction, E is young's modulus and ν is Poisson's ratio, subscript '*m*' refers to metal and '*c*' refers to ceramic. x is non-dimensional radius that is *r/a* and k is the ratio of outer diameter to inner diameter.

In model 1, Poisson's ratio is taken constant (0.333) and young's modulus vary according to equation (33) taking $n = 1$ while in model 2, both Poisson's ratio and young's modulus vary according to equation (32) and equation (33) taking $n = k = 2$. Material properties of the metal and ceramic used are: $E_m = 200$ *GPa*, $E_c = 360$ *GPa*, $\nu_m = 0.333$, $\nu_c = 0.2$ and the vessels are subjected to unit internal pressure that is 1 *GPa.*

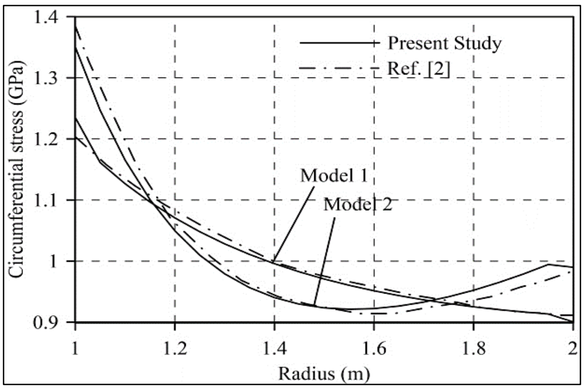

Fig. 2: *Validation of the Work.*

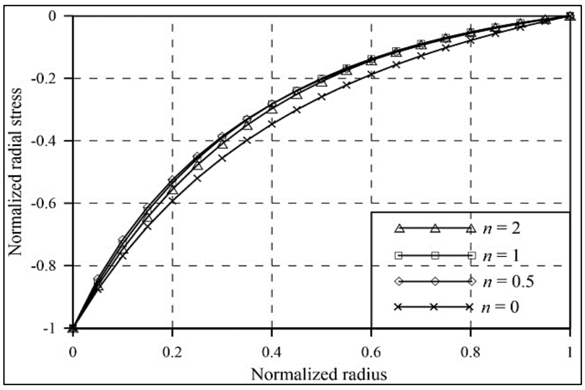

Fig. 3: *Normalized Radial Stress Distribution (Ceramic-Metal FGM).*

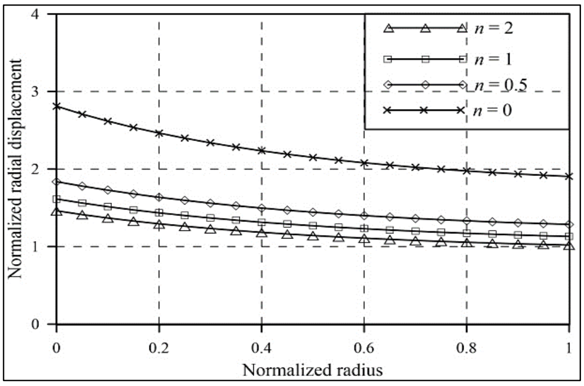

Fig. 4: *Normalized radial Deformation Distribution (Ceramic-Metal FGM)*

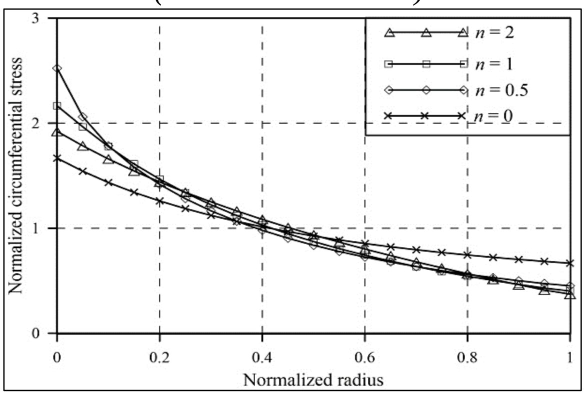

Fig. 5: *Normalized Circumferential Stress Distribution (Ceramic-Metal FGM).*

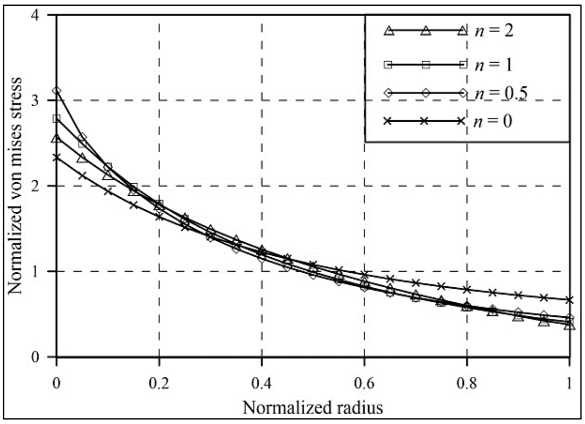

Fig. 6: *Normalized von Mises Stress Distribution (Ceramic-Metal FGM).*

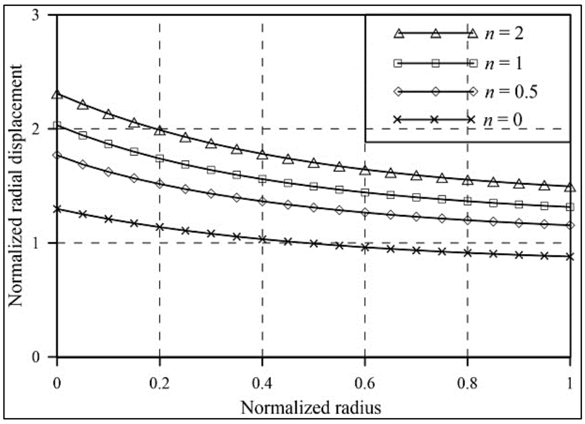

Fig. 7: *Normalized Radial Deformation Distribution (Metal-Ceramic FGM).*

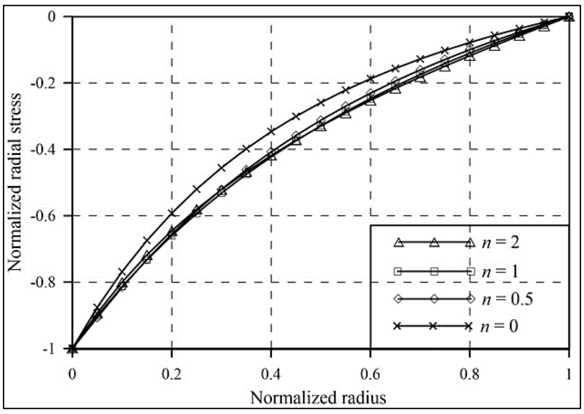

Fig. 8: *Normalized Radial Stress Distribution (Metal-Ceramic FGM).*

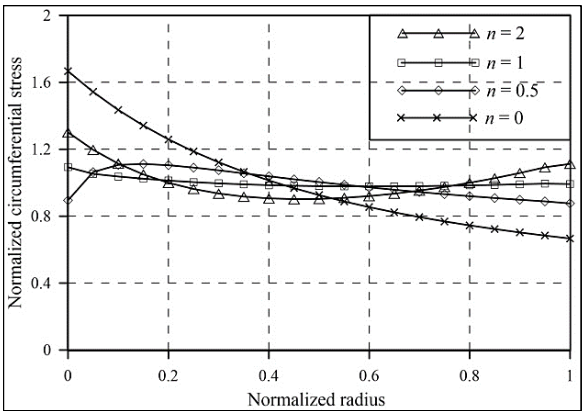

Fig. 9: *Normalized Circumferential Stress Distribution (Metal-Ceramic FGM).*

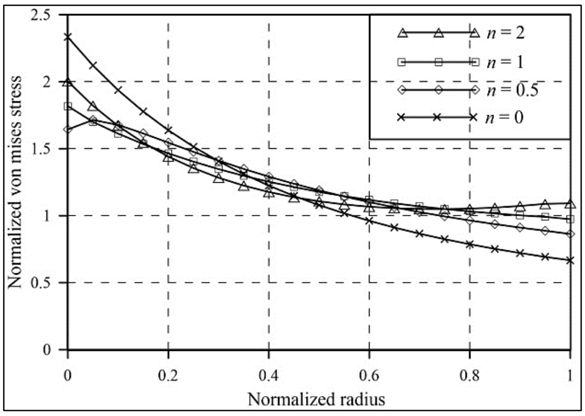

Fig. 10: *Normalized von Mises Stress Distribution (Metal-Ceramic FGM).*

Numerical Results

In this section, some numerical problems of rotating FGM cylindrical pressure vessels are analyzed by finite element method and the effect of grading parameter *n* on stress and deformation states is find out. Vessels have same geometric parameters as above and rotating with an angular velocity of 200 *rad/s*, subjected to unit internal pressure. Material is model by Mori-Tanaka scheme for which properties used are as [10]: E_m = 70 *GPa*, E_c = 151 *GPa*, B_m = 58.333 *GPa*, B_c = 128.333 *GPa*, G_m = 26.9231 *GPa*, G_c = 58.0769 *GPa* and ν = 0.3, where, *B* and *G* denotes bulk modulus and shear modulus.

Figure 3–6 show the distribution of radial deformation, radial stress, circumferential stress and von Mises stress respectively in a normalized scale along radial direction for ceramic-metal FGM vessels while Figure 7–10 show the same for metal-ceramic vessels. Stresses are normalized by dividing to the internal pressure and deformation is normalized as:

$$\bar{u} = \frac{u_r}{a} \times 100 \tag{34}$$

It is observed that stresses and deformation both are maximum at inner radius and minimum at outer radius for all cases. n = 0 indicates unit volume fraction of the outer material means a homogeneous shell. In ceramic-metal FGM, shell having n = 0 (metallic shell) has the maximum radial deformation while shell having n = 2 has minimum radial deformation. It is observed that radial deformation decreases with an increase in grading index n in ceramic-metal shell. Increasing n means volume fraction of the outer material (metal) is decreasing and inner material (ceramic) is increasing, which reduces radial deformation. Radial stress is unit at inner radius (equal to applied pressure) and zero at outer radius for all cases, which confirms the boundary conditions applied. Negative sign indicates compressive radial stress throughout the radius. From Figures 3 and 7, it can be seen that radial stress is very less affected by grading parameter n. Complete metallic shell (n = 0) has lowest radial stress while FGM shell having n = 1 has the highest radial stress. Circumferential stress as well as von Mises stress both are highest for n = 0.5 and decreases with increasing grading index but are minimum for n = 0 in ceramic-metal FGM.

In case if metal-ceramic shell n = 0, i.e., homogeneous ceramic shell has lowest radial deformation and highest radial, circumferential as well as von Mises stress. Radial deformation increases and radial stress decreases with increasing grading index n, due to increasing metallic content in FGM. Circumferential stress as well as von Mises stress both are lowest for n = 0.5 and increases with increasing grading index n, but complete ceramic shell (n = 0) has the maximum circumferential and von Mises stress in metal-ceramic FGM.

By comparing all grading index in ceramic-metal and metal-ceramic FGM vessels, it can be seen that metal-ceramic FGM shell having n = 0.5 has the lowest radial, circumferential and von Mises stress, therefore it is suggested that to optimize strength to stress ratio in rotating cylindrical pressure vessel, metal-ceramic FGM shell having n = 0.5 is most suitable.

CONCLUSION

The present work proposes a study of rotating FGM cylindrical pressure vessel using finite element method. Functionally gradation of the material properties is achieved by Mori-Tanaka scheme. Material properties of aluminum metal and zirconia ceramic are used and metal-ceramic as well as ceramic-metal both the type of FGM is considered. Principle of stationary total potential (PSTP) is used for finite element formulation. The results obtained are found to be in good agreement with established reports. Further, it is observed that there is a significant reduction in stresses and deformation behavior of the FGM shell as compared to homogeneous shell. It is suggested that metal-ceramic FGM shell having n = 0.5 is most efficient for the purpose of rotating cylindrical pressure vessel among all other FGMs investigated.

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