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An Analytical Model for Sub grid Force-Equilibrium Approach in Submerged Vegetative Channel

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ABSTRACT

A new, analytical model of the vertical flow velocity profile of submerged vegetation has been developed. Velocity profile can be divided into three zones: linear zone which is closed to bed with linear velocity distribution, logarithmic zone which involves the main channel without vegetative cover and the middle zone (transition zone) that is affected by the momentum of upper zone. For the surface layer, Prandtl's mixing length concept is adopted resulting in the well known logarithmic flow velocity profile. A new virtual roughness height concept is introduced for both logarithmic and vegetative zones, which accounts for the effect of vegetation on velocity distribution in these zones. From the continuity condition that both actual value and gradient of the flow velocity of the vegetation and the surface layer should be equal at the interface ($z=h_p$), velocity at the interface is obtained. This value is used as a reference value for obtaining velocity profile in vegetation zone using sub grid force equilibrium approach.

Keywords: *Vegetated open channel flow, Force equilibrium approach, Velocity Distribution*

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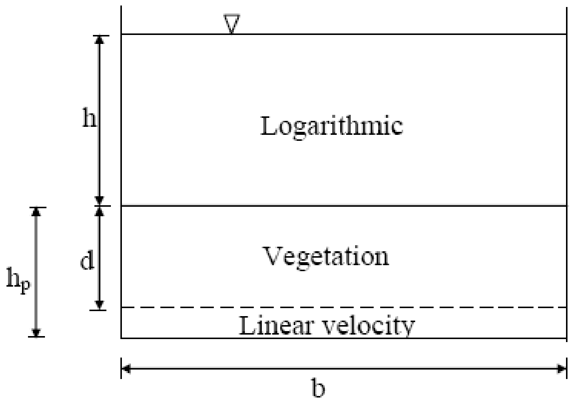
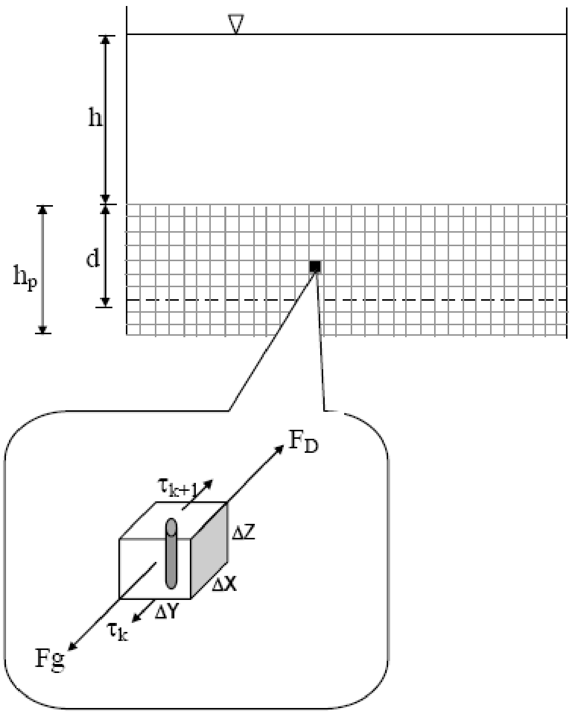
INTRODUCTION

Computation of velocity distribution and discharge capacity in vegetated waterways is central to many river and flood protection-engineering projects. Vegetation in river and channel banks alters velocity distribution in vegetated zones of these waterways. Additional flow resistance due to drag force induced by vegetation canopy is responsible for energy losses. Many theories were put forward for computation of velocity distribution in vegetated zone, The divided channel Method (DCM) which was proposed by Lotter (1933) acknowledges the lateral variation in boundary roughness, flow depth and hence velocity across the river /floodplain section. The researchers (e.g. Kaiser, 1984; Pasche, 1984; Bertram, 1985; Mertens, 1989; Nuding, 1991) latter modified DCM for use in partially vegetated channels. Empirical n-UR method which assumes unique relation between particular type of vegetation and Manning–Strickler roughness coefficient, was proved to be invalid by Kouwen and Li (1980) when the vegetation is short and stiff and slope is smaller than 0.05. Petryk and Bosmajian (1975) used force equilibrium approach for emergent plant conditions. They equated gravitational force with two resistant forces namely shear force and drag exerted by vegetation. The German Association for Hydraulic Engineering (DVWK, 1991) introduced hydraulic resistance method based on vegetation density. Researchers (e.g. Kouwen et al., 1969; Tsujimoto et al., 1992; Nepf and Vivoni, 1999) introduced log law for vegetated layer using undeflected plant height.

Burke and Stolzenbach (1983), Nakagawa et al. (1992), Shimizu and Tsujimoto (1994) and Lopez and Garcia (1997) used two-equation turbulence model for multi dimensional flow problems. This was first introduced by Wilson and Shaw (1977), with drag sink terms in momentum and turbulent transport equation.

THE ANALYTICAL MODEL





Vegetation Density (λ)

$$\lambda = \frac{\text{Projected area of plant}}{\text{Total Volume}}$$

$$= \frac{d_p \ \Delta z}{\Delta x \ \Delta y \ \Delta z}$$

$$= \frac{d_p}{\Delta x \ \Delta y}$$

Force Equilibrium Approach: Forces acting on cubical water element are as follows,

1) Body force (component of self weigh):

$$F_G = \rho \ g \ S \ (\Delta x \ \Delta y \ \Delta z) \qquad (1)$$

2) Shear Force (F_S)

$$F_S = \tau_{k+1} (\Delta x \ \Delta y) - \tau_k (\Delta x \ \Delta y)$$

$$F_S = (\tau_{k+1} - \tau_k) \ \Delta x \ \Delta y$$ OR
O

$$F_S = \frac{\tau_{k+1} - \tau_k}{\Delta z} \Delta x \Delta y \Delta z$$

$$F_S = \frac{\partial \tau}{\partial z} (\Delta x \Delta y \Delta z) \qquad (2)$$

3) Drag Force due to vegetation (F_D):

$$F_D = \frac{1}{2} C_D \rho N A \overline{U}^2$$

Where,

N = No. of stems in a cell = 1.

A = Projected area of stem = $d_P \ \Delta z$

$\overline{U}$ = Average cell velocity (Assumed constant for small values of ΔZ)

$$F_D = \frac{1}{2} C_D \ \rho \ 1 \ (d_p . \Delta z) \ \overline{U}^2$$ OR

$$F_D = \frac{1}{2} C_D \ \rho \ 1 \left(\frac{d_p}{\Delta x \Delta y} \right) \overline{U}^2 \ (\Delta x \Delta y \Delta z)$$ OR

$$F_D = \frac{1}{2} C_D \ \rho \ \lambda \ \overline{U}^2 \ (\Delta x \Delta y \Delta z) \qquad (3)$$

Now the gravitational force can be equated to two resistance forces, the drag force of the vegetal obstruction and the shear force on the cell.

$F_G = F_S + F_D$

$$\rho g S(\Delta x \Delta y \Delta z) = \frac{\partial \tau}{\partial z}(\Delta x \Delta y \Delta z) + \frac{1}{2} C_D \rho \lambda \overline{U}^2 (\Delta x \Delta y \Delta z)$$

$$\rho g S = \frac{\partial \tau}{\partial z} + \frac{1}{2} C_D \ \rho \ \lambda \ \overline{U}^2$$

$$\frac{\partial \tau}{\partial z} = \rho g S - \frac{1}{2} C_D \ \rho \ \lambda \ \overline{U}^2$$

VELOCITY DISTRIBUTION

Logarithmic Zone:

Velocity distribution in logarithmic zone is given by

$$\frac{U}{u^*_{hp}} = 2.5 \ \ln\left(\frac{y}{d_L}\right) + 5.5$$

where,

U^*_{hp} = Virtual bed shear velocity

y = distance measured from bed.

d_L = Virtual Roughness height

Virtual Roughness height can be calculated for the logarithmic zone using Prandtl's mixing length theory. Prandtl's mixing length ($lm_)$ can be taken as product of Von Karmans constant and Virtual Roughness height.

Lm_L = k x d_L

For Logarithmic zone,

$$Lm_L = L\left[0.14 - 0.08\left(1 - \frac{h_p}{L}\right)^2 - 0.06\left(1 - \frac{h_p}{L}\right)^4\right]$$

where,

L = b/2

Vegetated Zone

Interface (top of vegetation) is the last layer of the logarithmic zone. Although Velocity of this layer is calculated using logarithmic velocity profile, as given below, corrected value of virtual Roughness height is used to account for effect of vegetation.

Velocity at the interface is given by,

$$\frac{U_{hp}}{u^*_{hp}} = 2.5 \ \ln\left(\frac{h_p}{d_v}\right) + 5.5$$

where, virtual Roughness height (d_v) is given by ,

lm_v = k x d_v

and

$$lm_v = L\left[0.14 - 0.08\left(1 - \frac{h_p}{H}\right)^2 - 0.06\left(1 - \frac{h_p}{H}\right)^4\right]$$

Velocity at the interface is used as a reference value to obtain velocity distribution in vegetated zone. Vegetated zone is divided in two numbers of layers each of thickness Δz. velocity of each layer is obtained by solving force equilibrium equation **(4)** for τ numerically.

Shear stress distribution

Shear stress of the layers in logarithmic zone,

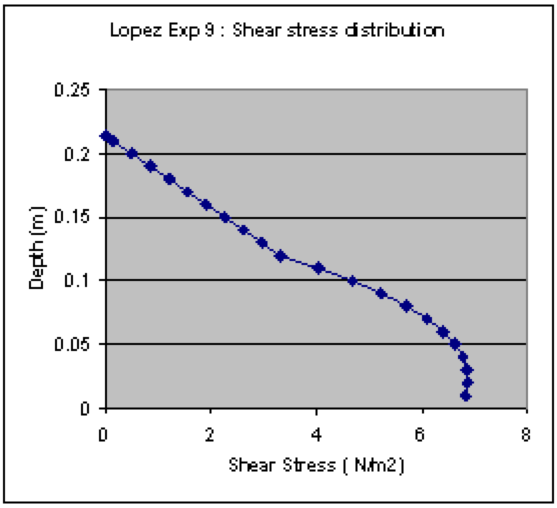
$\tau = \rho g y S$

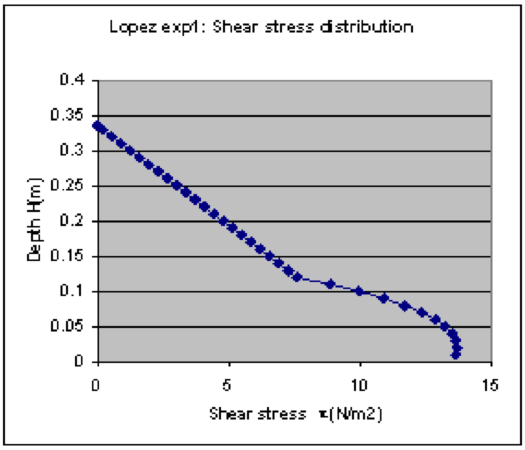
Shear stress at the interface,

$$\tau \;=\; \rho g h S$$

Shear stress for the layers of vegetated zone is calculated by equation,

$$\tau = \rho \;\; l_m^2 \left(\frac{\partial U}{\partial z} \right)^2$$





APPLICATION OF ANALYTICAL MODEL TO LABORATORY EXPERIMENTS

Tsujimoto et al and Lopez conducted experiments in a rectangular flume and measured vertical velocity distribution in stream wise direction. For all experiments the drag coefficient was determined to be equal to 1.0 for the given Reynolds numbers. Different vegetative densities (λ), different vegetative element diameter (dp), and different vegetative heights (hp) were used.

Validation of analytical model for Lopez

	Tsusjimoto		Lopez	
Experiment	R32	A31	Exp1	Exp9
Geometry				
Type	rectangular flume		rectangular flume	
Length L [m]	n/a		19.50	
Width W [m]	n/a		0.91	
Depth H [m]	0.0747	0.0936	0.335	0.214
Slope I [$*10^{-3}$]	2.13	2.60	3.60	
Vegetation				
Type	fully vegetated, submerged		fully vegetated, submerged	
Vegetation Height h [m]	0.041	0.046	0.12	
Plant Diameter D [$m*10^{-3}$]	1	1.5	6.4	
Plant density λ [1/m]	10	3.75	1.09	2.46
Measuring Technique	hot-film anemometry micro propeller		acoustic doppler velocimetry	
Simulations				
No. of Cells	60,000	15,000	12,000	3,000
Sensitivity Simulations				
No. of Cells	4,576	-	3,000	-
No. of Cells	-	-	-	-

Exp 1:

The vegetated zone is divided in to 12 strips giving,

$\Delta z = 0.010$

Prandtl's Mixing Length (Lm_v),

$$Lm_v = L\left[0.14 - 0.08\left(1 - \frac{h_p}{H}\right)^2 - 0.06\left(1 - \frac{h_p}{H}\right)^4\right]$$

$Lm_v = 0.044075$

Roughness height (d_V),
$l_m = kd$

$d = 0.1101875$

Velocity at the Interface,

$$U_{hp} = u_{hp}^{*}\left[2.5 \;\; \ln\left(\frac{y}{d}\right) + 5.5\right]$$

$U_{hp} = 0.497838$ m/s

Shear stress at the top of vegetation,

$\tau_{hp} = \rho ghS = 7.593$ N/m^2

Now applying Force equilibrium equation for any cell at the surface layer, assuming the velocity to be constant for entire cell height, for small value of Δz. i.e taking $\overline{U} = U_{hp}$

$$\frac{\partial \tau}{\partial z} = \rho g S - \frac{1}{2} C_D \quad \rho \quad \lambda \quad \overline{U}^2$$

$$\frac{7.593 - \tau_{11}}{0.01} = 1000 x 9.81 x 3.6 X 10^{-3}$$
$$- \frac{1}{2} x1x1000x1.09x0.497838^2$$

$\tau_{11} = 8.590859$ N/m^2

Now, velocity at this level (considering actual velocity gradient),

$$\tau = \rho \quad l_m^2 \quad \left(\frac{\partial U}{\partial z}\right)^2$$

$$8.590859 = 1000 \text{ x } (0.044075)^2 \text{ x } \left(\frac{\partial u}{\partial z}\right)^2$$

$$\left(\frac{\partial u}{\partial z}\right) = 2.102935$$

$$\frac{0.497838 - U_{11}}{0.01} = 2.102935$$

$U_{11} = 0.476871$

Velocity for the other strips is tabulated in Table I.

Velocity distribution for logarithmic zone can be obtained from log law, and is tabulated in Table II.

Table I Velocity Distribution for Vegetated Zone (Lopez Exp1) Zone

y	τ	du/dz	U
0.12	7.59294	1.97703	0.4979
0.11	8.590859	2.102939	0.476871
0.1	9.477059	2.208743	0.454783
0.09	10.25111	2.297173	0.431811
0.08	10.91416	2.370301	0.408108
0.07	11.46871	2.429773	0.383811
0.06	11.9184	2.47695	0.359041
0.05	12.2678	2.512995	0.333911
0.04	12.5223	2.538928	0.308522
0.03	12.6879	2.555661	0.282965
0.02	12.77112	2.564028	0.257325
0.01	12.77884	2.564803	0.231677

Table II Velocity Distribution for Logarithmic Zone (Lopez Exp1) Zone.

y	U/U*	U
0.13	6.423065	0.559642
0.14	6.608335	0.575784
0.15	6.780817	0.590813
0.16	6.942163	0.604871
0.17	7.093725	0.618076
0.18	7.236621	0.630527
0.19	7.371789	0.642304
0.2	7.500022	0.653477
0.21	7.621997	0.664105
0.22	7.738298	0.674238
0.23	7.849427	0.683921
0.24	7.955826	0.693191
0.25	8.057881	0.702083
0.26	8.155933	0.710626
0.27	8.250284	0.718847
0.28	8.341203	0.726769
0.29	8.428931	0.734413
0.3	8.513685	0.741797
0.31	8.595659	0.74894
0.32	8.675031	0.755855
0.33	8.75196	0.762558
0.335	8.789555	0.765834

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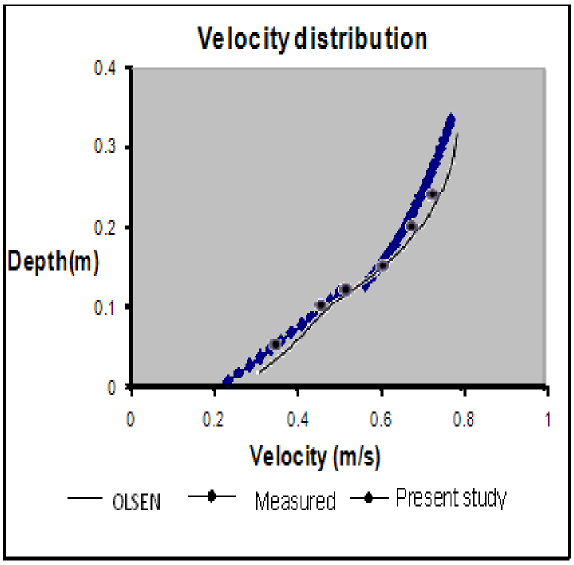


Fig 1 Calculated and Measured Width Averaged Velocity Profiles of Lopez's Exp 1.

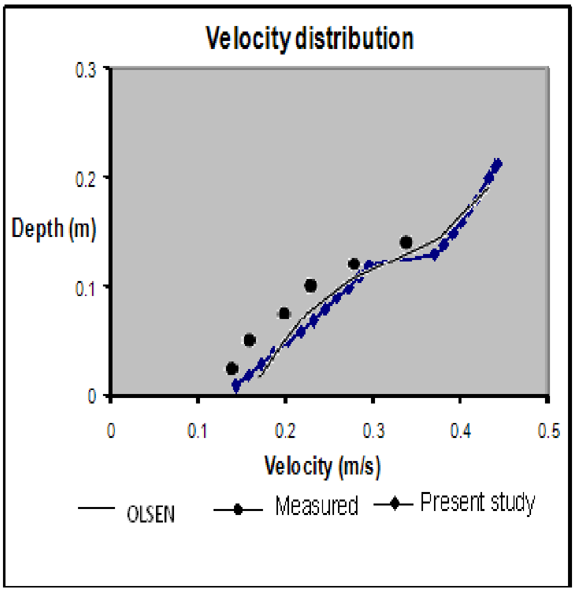


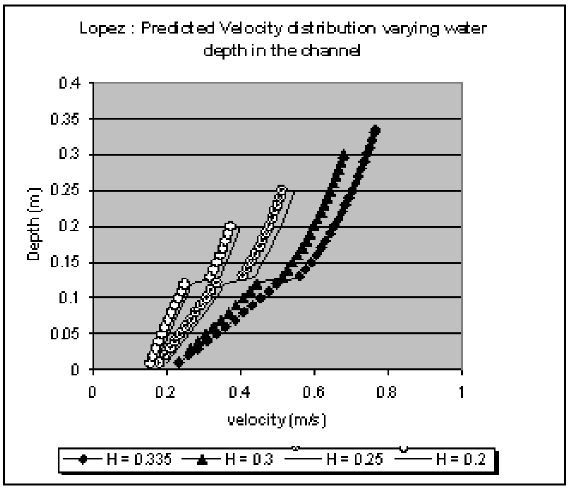
Fig 2 Calculated and Measured Width Averaged Velocity Profiles of Lopez's Exp 9.

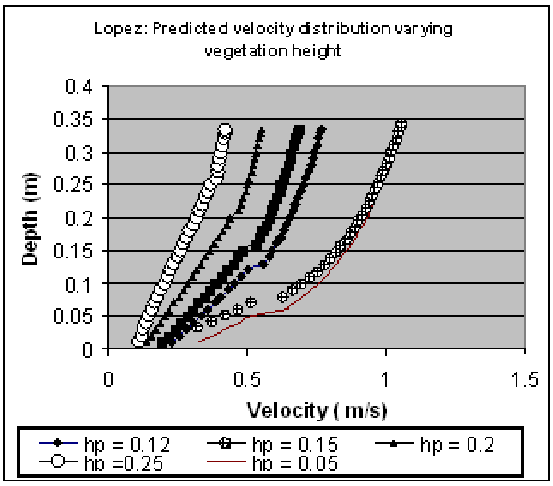
Table III Velocity Distribution for Vegetated Zone (Lopez Exp 9) Zone.

y	τ	du/dz	U
0.12	3.319704	1.035164	0.295564
0.11	4.041048	1.142106	0.284143
0.1	4.680956	1.229211	0.271851
0.09	5.236801	1.300147	0.258849
0.08	5.707778	1.357353	0.245276
0.07	6.094589	1.402592	0.23125
0.06	6.39919	1.437215	0.216878
0.05	6.624573	1.462306	0.202255
0.04	6.774568	1.478768	0.187467
0.03	6.853678	1.487377	0.172593
0.02	6.866916	1.488813	0.157705
0.01	6.819668	1.483682	0.142868

Table IV Velocity Distribution for Logarithmic Zone (Lopez Exp 9) Zone.

y	τ	U/U*	u
0.13	2.966544	6.423065	0.370078
0.14	2.613384	6.608335	0.380752
0.15	2.260224	6.780817	0.39069
0.16	1.907064	6.942163	0.399987
0.17	1.553904	7.093725	0.408719
0.18	1.200744	7.236621	0.416952
0.19	0.847584	7.371789	0.42474
0.2	0.494424	7.500022	0.432129
0.21	0.141264	7.621997	0.439157
0.214	0	7.669169	0.441874





CONCLUSION

Prediction of velocity profile for vegetated channel is important to hydraulic engineers. The analytical model presented in this study is a quick hand calculation method for obtaining velocity distribution. The model is validated for the Lopez's Experiments and is found to give fairly accurate results. The model is also capable of predicting reverse flow in case of low values of Logarithmic zone thickness. The model is not tested for emergent Vegetated channel due to lack of data. The model can be further modified considering the virtual bed little below the top of vegetation based on flexibility studies of vegetation type, and finding distance of the virtual bed from interface. More accurate results can be obtained by finding prandtls mixing length for different layers, to account for the effect of vegetation on individual layers, instead of different zones. Also Prandtl's mixing length should be suitably altered for over submerged vegetated flows.

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