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Sensitivity of Rigid Pavement Responses to Pavement Layer Thickness Due to Wheel Load: A Nonlinear Finite Element Study

Md. Mizanur Rahman, **Md. Newaz Murshed***
Civil Engineering Department, Bangladesh University of Engineering and Technology
Dhaka, Bangladesh

Abstract
The behavior of a jointed plain concrete pavement (JPCP) has been investigated under single wheel load for interior loading using finite element technique to predict the critical pavement responses for both linear and nonlinear geometrical characterization. The idealized pavement system is analyzed using 3D finite element analysis with the general purpose finite element software ABAQUS. The developed 3D model was analyzed for four combinations of material characterizations- (1) linear base and subgrade material, (2) nonlinear base and linear subgrade, (3) linear base and nonlinear subgrade, and (4) nonlinear base and nonlinear subgrade. Effects of different pavement layer thicknesses i.e. slab thickness and base course material thicknesses on the critical pavement responses were studied. The study shows that for a jointed plain concrete pavement, the maximum values of pavement deflection of slab top, tensile stress at the bottom of concrete slab and vertical pressure on top of subgrade are reduced significantly up to a concrete slab thickness of 225 mm (9 inch), above which the influence of slab thickness on pavement responses reduces significantly. The effects of base course material thickness on the maximum values of pavement deflection, tensile stress and subgrade pressure are less significant compared to effects of slab thickness.

Keywords: *Jointed Plain Concrete Pavement, Pavement Responses, Nonlinear Geomaterial, Finite Element Analysis, Abaqus*

****Author for Correspondence*** E-mail: mizanur@ce.buet.ac.bd

INTRODUCTION

Rigid pavements are one of the principal kinds of pavement widely in use throughout the world for both roadways and runways. But the design procedures available today are mostly based on empirical equations dependent pavement analysis results. With the advent of computer and the versatile finite element method, numerical analysis is gaining popularity day by day over empirical methods of pavement analysis to determine the critical pavement responses. Research works has been carried out through the globe to predict the critical pavement responses using versatile finite element analysis software widely available today.

The design of pavements has evolved greatly but still empiricism plays a very important part in the analysis of pavements. The formulas Westergaard [1] developed originally considered only a single wheel load with a circular, semicircular, elliptical, or semi-elliptical contact area. Whereas, the influence charts developed by Pickett and Ray [2] can be applied to multiple-wheel loads of any configuration. Both the formulas and the influence charts are applicable only to a large slab on a liquid foundation.

Finite element methods for analyzing slabs on elastic foundations of both liquid and solid types were developed by Cheung and Zienkiewicz [3]. These methods were applied to jointed slabs on liquid foundation by Huang and Wang [4] and on solid foundations by Huang [5]. In Collaboration with Huang, Chou [6] developed finite element computer programs named WESLIQID and WESLAYER for the analysis of liquid and layered foundations, respectively. Other available finite element computer programs include ILLI-SLAB developed by the University of Illinois, JSLAB developed by the Portland Cement Association (PCA) and RISC developed by Resource International, Inc. ILLISLAB was originally developed in 1977 for the structural analysis of one or two layers of slabs with or without mechanical load transfer system at joints and cracks. Heinrichs et al. [7] compared several available computer models for rigid pavements and concluded that both ILLI-SLAB and JSLAB, which is a similar finite element program, developed by PCA, were efficient to use and could structurally model many key design factors of importance. They also indicated that the ILLISLAB had extensive checking, revisions, and verification by many researchers and was free of errors than any other available program. The KENSLABS computer program developed by Huang [8] is based on finite element method, in which the slab is divided into rectangular finite elements with large number of nodes. Both wheel loads and subgrade reactions are applied to the slab as vertical concentrated forces at the nodes. The program was designed to analyze slabs on liquid, solid or layer foundations. All these three foundation types considered subgrade, subbase and base to be linearly elastic. Shaikh [9] also carried out a behavioral study of rigid pavement section using general purpose finite element software ANSYS. The model was generated directly using ANSYS CAD modeler. The surface layer is simulated through a two-dimensional plane surface while the subbase and subgrade are considered as elastic, homogeneous and linear springs supporting the top surface and are been modeled through one individual spring using effective or composite stiffness. He concluded that non linear properties of both concrete and foundation material can be used in order to better simulate the real field condition.

Development of the 3D Finite Element Model

The finite element modeling of this study consists of three components – the jointed plain concrete pavement surface, the granular base course material, and the subgrade soil. It also encompasses the interfaces between any two of these three components of the pavement system. In Figure 1, a schematic diagram of a jointed plain concrete pavement system has been shown.

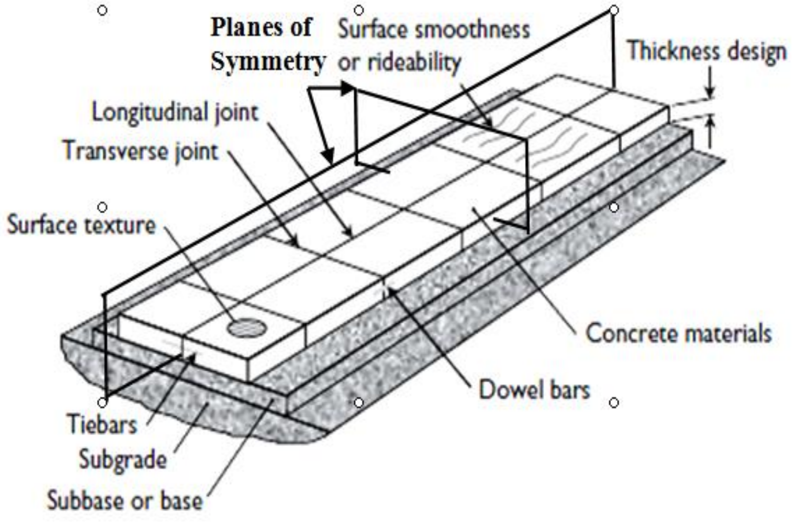

Fig. 1: *Schematic Diagram of a Jointed Plain Concrete Pavement.*

The pavement system has two vertical planes of symmetry as shown in Figure 1, one along the center line of pavement and another across the center line of the pavement. These planes of symmetry provide an inherent benefit in the finite element modeling and analysis of the pavement system. Because of these symmetries, only one quarter of the pavement is sufficient for finite element modeling, which requires less computing capacity with a consequent save in analysis time.

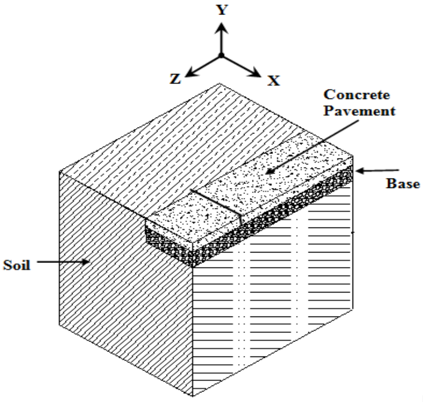

Fig. 2: *Schematic View of Quarter of the Jointed Concrete Pavement System.*

Figure 2 shows a schematic diagram of a one quarter of the jointed plain concrete pavement (JPCP) that will be used for developing the present 3D FE model. The concrete slab is considered to be laid on a granular base course material instead of being placed directly on the natural subgrade soil. Soil is considered to extend horizontally across the pavement beyond the base course material and concrete slab edge. In the Figure 2, X direction corresponds to the width of the pavement and Z direction corresponds to the length of the pavement in the direction of traffic movement and Y direction corresponds to the depth of the pavement.

Modeling of Soil

Characterization of soil is a complex phenomenon because of its nonlinearity and several other factors such as its interaction with structures and time dependent effects- creep, temperature and load history. Due to the orthotropic nature of soil, selection of proper element and material property inputs are very important to simulate the actual condition. Soil was modeled using C3D8 element. C3D8 is 8-node linear brick used for the three-dimensional modeling of solid structures. The element is defined by eight nodes. Each node of the element has three degrees of freedom at each node: translations in the nodal x, y, and z directions. Soil is modeled as an elastoplastic isotropic material. So elastic property and plastic property have to be defined. Elastic property is defined as a linear elasticity based on elastic modulus (E) and Poisson's ratio (v). But defining plastic property of a soil is rather complex. In this study Modified Drucker-Prager Cap model is selected for soil plasticity.

Modeling of Granular Base

Granular material characterization is also a complex phenomenon because of its nonlinear stress dependent behavior and several other factors. The cross-anisotropic, or laterally isotropic, behavior is a response that is particular to unbound granular materials. The granular base was modeled using the same C3D8 element. In this study Extended Drucker-Prager model is selected for granular base plasticity.

Modeling of Concrete Slab

The concrete slab was modeled using C3D8 element. For this study, concrete is considered to be homogenous and linearly elastic following Hooke's law. The input parameters are modulus of Elasticity, $\boldsymbol{E}$ and Poisson's ration $\boldsymbol{\upsilon}$.

Structural Idealization of the Pavement System

For analyzing the pavement system it is required to idealize the physical system. A typical two-lane jointed plain concrete pavement slab with lane width of 3.5 meter (12 feet) each was selected and one lane was considered for developing the model for analysis. A joint spacing of 3.5 meter (12feet) was selected for transverse joints for this study. For the purpose of simplicity, one quarter of the pavement system was analyzed as it has two axis of symmetry, one along the pavement and another across the pavement as shown in Figure 1 and Figure 2. Depth of the subgrade soil and its extension in the horizontal direction was found out through sensitivity analysis. No shoulder, either concrete or flexible, was considered in this study. Natural subgrade was considered beyond the boundary of concrete slab and granular base in the horizontal direction. Smooth boundary conditions were applied along the bottom and the side faces of the boundary of the model which is described in the preceding paragraphs. The objective of using smooth boundary conditions is to make the system as flexible as possible. Here, the bottom surface as well as all other vertical sides was considered to be on rollers so that no rigid body motion takes place. Relative displacements were allowed at the interfaces of concrete with granular base and soil but no relative displacement was allowed between granular base and soil interfaces. Interaction with adjacent slabs (i.e. transfer of load and deflection along pavement slab joints) was not considered in this analysis.

Determinations of Material Properties and Input Parameters

In order to study in detail the behavior of jointed plain concrete pavement to find out the critical pavement responses the reference model has been chosen and analyzed. The pavement geometries and elastic material properties for the reference model are provided in Table 1.

Table 1: *Pavement Geometry and Material Properties.*

Layer	*Thickness/ Depth (m)*	*Modulus of Elasticity, Es (MPa)*	*Poisson's Ratio, υ*	*Unit Weight Kg/m3*
Concrete surface	0.225	25000	0.2	2400
Granular base	0.30	120	0.3	2147
Subgrade soil	24	28	0.3	1920

The jointed plain concrete pavement is composed of three different material type-concrete slab, base or subbase aggregate (if used) and subgrade soil. For the present study, concrete is assumed to be linearly elastic, homogenous and isotropic. The base course aggregate and the subgrade soil are considered to exhibit stress dependent nonlinear behavior.

Granular Base

Availability of strength and deformation characteristic data for aggregate layer materials is a great concern for proper modeling of these materials. For the present study, test data from the Minnesota Road Research Project (Mn/ROAD) were used. The Minnesota Department of Transportation in cooperation with the Federal Highway Administration and the Local Roads Research Board of Minnesota constructed the project.

Soil

The data used for determining the soil properties were collected from the detailed results of geotechnical investigation section of the final report on the Paisarhat Bridge Over Pisa River at Barisal, Bangladesh. The investigation was conducted by the Civil Engineering Department under Bureau of Research Testing and Consultancy (BRTC) of Bangladesh University of Engineering and Consultancy.

Loading

As the effect of the load value, configuration and shape are not considered in this study; only a single wheel load of 40 KN (9,000 lb) of an equivalent 80KN (18,000 lb) single axle load is adopted. The stiffening effect of the tire wall is neglected, hence, the contact pressure on the road is assumed to be equal to the tire pressure. Furthermore, the contact pressure distribution is assumed uniform and is taken as 550 KPa (80 psi) which was applied over a circular area of 152 mm (6inch) at the center of the concrete slab.

Soil Extension

A significant number of finite element analyses were performed changing the depth of the native soil layer i.e. subgrade. For the reference model, the thickness of the concrete slab and granular base was taken to be 225 mm (9 inch) and 300 mm (12 inch) respectively. No shoulder was considered for the present study. Instead, soil was considered to extend horizontally across the pavement beyond the concrete slab.

For the purpose of simplicity, horizontal extent of subgrade soil was also changed based on the approximation of stress distribution in the ration of (1 H: 1 V) i.e. the horizontal extent was considered to be half of the corresponding vertical extent of the subgrade soil for the one quarter of the pavement. A study on the effect of site soil extent on the maximum deflection (δ_c) of the top of the concrete slab at the center of the circular loaded area and the maximum tensile stress (σ_i) at the bottom of the concrete slab are performed. From the study it is evident that the values of the parameters do not show any significant change beyond a depth of 24 meter (80 feet). Therefore, a subgrade depth of 24 meter was considered to be sufficient for further analysis.

Optimum Mesh Size

Main objective of this study is to examine the concrete pavement response i.e. deflection (δ_c) and tensile stress (σ_i) at the bottom of the concrete slab at the center point of loading. So deflection of the top surface of the concrete slab and tensile stress at the bottom of the slab at the center of the loaded area are considered as the prime variable for mesh sensitivity analysis. A study on the variation of deflection of top surface at the center point of circular loading and maximum tensile stress at the bottom of the concrete slab with different element sizes was performed and found that a total number of 53855 elements were good enough to get optimum result considering the tradeoff between computational time and accuracy.

Verification of Fe Model

An attempt has been made for the verification of the numerical modeling of the pavement system with the available numerical analysis results. Kim, *et al.* [10] carried out a linear elastic analysis of flexible pavement for 3D and axisymmetric modeling using the finite element software ABAQUS. He matched the results with the results provided by Huang (2004) obtained by the linear elastic layered program, KENLAYER.

The 3D model developed by Kim, *et al.* [10], had 15,168 20-noded hexahedron element and 67,265 nodes. All the vertical boundary nodes had roller supports with fixed boundary nodes used at the bottom. The wheel load was applied as a uniform pressure of 551 KPa (80 psi) over a circular area of 152 mm (6 inch) radius. Table 2 lists the three-layered conventional flexible pavement geometries and the material properties used in the 3D linear elastic FE analyses.

Table 2: *Pavement Geometry and Material Properties for Finite Element Model Verification.*

Layer	**Thickness (mm)**	**E or MR (MPa)**	***μ***
Asphalt concrete	76	2759	0.35
Base	305	207	0.40
Subgrade	20955	41.4	0.45

For the verification of the 3D modeling of this study, the flexible pavement analyzed by Kim, *et al.*, [10] was reconstructed in the ABAQUS environment using the same pavement geometries and material properties. Instead of 20-node hexahedron elements, 8-noded brick element was chosen which was used to develop the jointed plain concrete pavement for the present study.

The reason behind the choice of 8-noded brick element instead of 20-noded hexahedron element is that, 20-noded hexahedron element choice results in grater computing capacity and time than the 8-noded brick element. Predicted pavement surface deflection (δ_{surface}) and certain critical pavement responses, i.e., vertical stress and strain on top of subgrade (σ_v and ε_v) and horizontal stress at the bottom of the asphalt concrete layer (σ_h), are compared in Table 3 with the linear elastic KENLAYER closed-form solutions and results predicted by Kim. The results show in general a very good agreement with the KENLAYER result and results obtained by Kim, *et al.*

Table 3: *Comparison of Predicted Response for 3d Model with Results From (kim et al.)Study.*

Pavement Response (Tension is positive)	**KENLAYER**	**ABAQUS with 20-node hexahedron elements. (Kim., et al.)**	**ABAQUS with eight-node brick elements**	**Difference (%)**
δ_{surface} (mm)	- 0.927	-0.909	-0.913	+ 0.44
$\sigma_{h \text{ bottom of AC}}$ (MPa)	0.777	0.777	0.821	+ 5.6
$\sigma_{v \text{ top of subgrade}}$ (MPa)	-0.041	-0.040	-0.0365	-10
$\varepsilon_{v \text{ top of subgrade}}$ ($\mu\varepsilon$)	-936	-930	-845	-9.1

Pavement Responses to Wheel Load

Comparisons of 3D Linear and Nonlinear FE Analysis

Differences between the pavement responses for linear elastic and nonlinear material characterization for 3D finite element analysis were determined using the reference 3D finite element model developed in the previous chapter. Four different combinations of material characterizations were used all using linear elastic concrete material properties and the following pavement layer characterizations: (1) linear elastic; (2) nonlinear base and linear subgrade; (3) linear base and nonlinear subgrade; and finally, (4) nonlinear base and nonlinear subgrade. Table 4 gives detailed comparisons of the predicted critical pavement responses.

Table 4: *Predicted Pavement Responses for 3d Analysis.*

Pavement Responses	**Linear Elastic**	**Nonlinear base and linear subgrade**	**Linear base and nonlinear subgrade**	**Nonlinear base and nonlinear subgrade**
Maximum surface deflection at slab top, $\delta_{surface}$ (mm)	-0.394	-0.394	-0.443	-0.443
Maximum tensile stress at slab bottom, σ_i (MPa)	0.95001	0.95001	0.95001	0.95001
Vertical compressive stress on top of subgrade, σ_v (Kpa)	-4.994	-4.994	-4.994	-4.994

The results of the 3D finite element analysis for the jointed plain concrete pavement due to applied wheel load are plotted in (Figure 3–5) for the four combinations of pavement material characterization.

In Figure 3, variation of vertical surface deflection of the top surface of the concrete slab is plotted with respect to distance from the centreline of the circular loaded area along the longitudinal direction of the pavement. The maximum vertical surface deflection of the concrete slab at the center of the circular wheel load is 0.394 mm for linear elastic analysis and 0.443 mm for nonlinear finite element analysis and decreases towards the edge of the concrete slab along the pavement direction. The maximum deflection predicted for nonlinear analysis is 12.4 % greater than that predicted for linear elastic analysis. Consideration of nonlinearity of base course material has significantly no effect on the pavement surface deflection as shown in Figure 3 .This is due to the fact that the base course material behaves as a linearly elastic material under the present loading condition. On the contrary, the subgrade soil settles more when material nonlinearity is considered.

Variation of the tensile stress developed at the bottom surface of the concrete slab along the longitudinal direction of the pavement is shown in Figure 4. The maximum tensile stress developed at the bottom of the concrete slab predicted for the linear elastic and nonlinear elastic finite element analysis have the same value of 0.95 MPa (138 psi). The value of the tensile stress at the bottom of the concrete slab is maximum below the center of the circular wheel load and it decreases towards the edge of the pavement. Although the deflection predicted for nonlinearity of subgrade soil is greater than linear elastic condition the tensile stress at the bottom surface of slab has no difference. This phenomenon can be explained by the fact that the relative deflection of the center of the slab with respect to its corner is same resulting in the same bottom surface tensile stress.

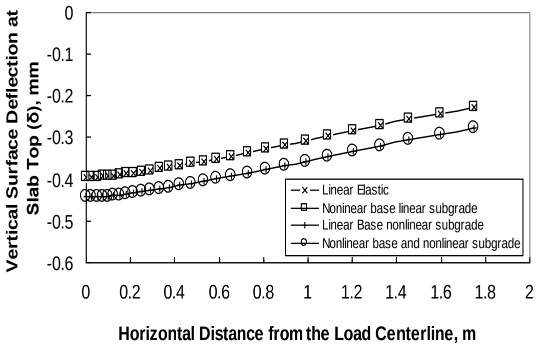

Fig. 3: *Vertical Surface Deflection Along Longitudinal Direction of Pavement for the 4 Combinations of Materials Characterization.*

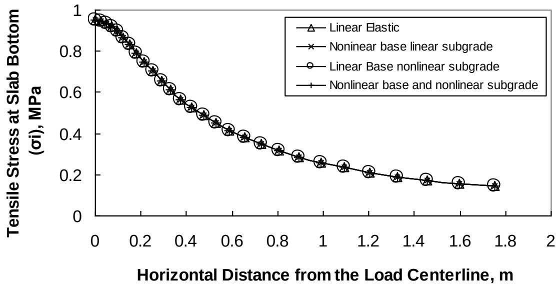

Fig. 4: *Tensile Stress at Slab Bottom along Longitudinal Direction of Pavement for the 4 Combinations of Materials Characterization.*

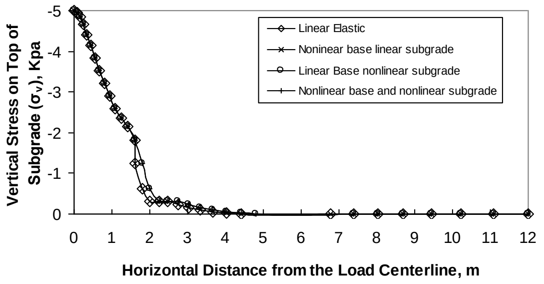

Fig. 5: *Vertical Stress on Slab top Along Longitudinal Direction of Pavement for the 4 Combinations of Materials Characterization.*

The variation of vertical compressive stress on the subgrade surface along the longitudinal direction of the pavement is plotted in Figure 5 for the four material characterization combinations. The maximum values of the vertical compressive stress on the top surface of the subgrade soil predicted by both linear elastic and nonlinear finite element analysis have been found to be 4.994 KPa. The compressive stress on subgrade soil surface is maximum below the center of circular wheel load and decreases towards the edge of concrete slab. The concrete slab acts as a rigid plate under the applied wheel load and distributes the load over a greater surface area of the underlying base and subgrade area thus minimising the effect of stress concentration and localized large deflection.

Effect of Concrete Slab Thickness

A whole series of 3D finite element analysis were performed using the reference model to observe the effect of concrete slab thickness on the pavement responses. A reasonably wide range of concrete slab thickness was considered, starting from a considerably thin section of 100 mm (4 inch) thick to a large section of 350 mm (14 inch). For different values of slab thickness other geometrical properties of the reference model were kept constant. The effect of concrete slab thickness on the pavement responses are shown in Figure 6–8.

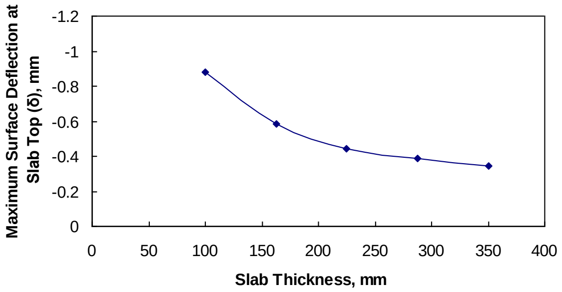

Fig. 6: *Variation of Vertical Surface Deflection of Slab Top with Slab Thickness.*

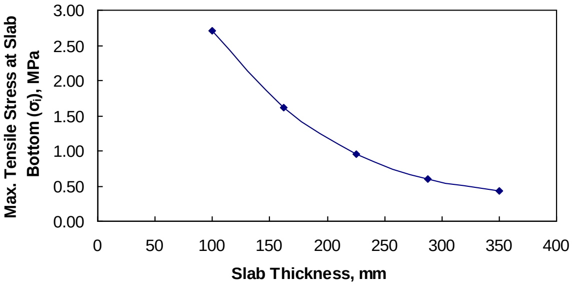

Fig. 7: *Variation of Maximum Tensile Stress at Slab Bottom with Slab Thickness.*

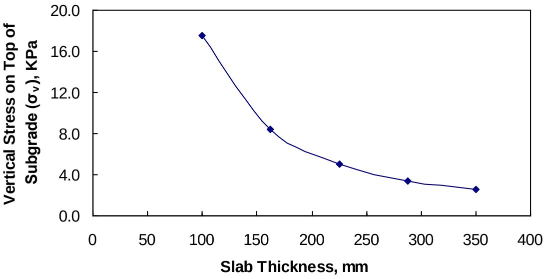

Fig. 8: *Variation of Compressive Stress on Top of Subgrade with Slab Thickness.*

Figure 6 shows the influence of slab thickness on the vertical deflection ($\delta_{surface)}$ of the top of concrete surface below the center of the circular loaded area. Figure 6 distinctly illustrates that vertical deflection of the top concrete surface decreases significantly with the increase of slab thickness up to a thickness of 225 mm (9 inch). Above this thickness of 225 mm, the influence is reduced. This behaviour is of concrete slab is easily perceivable, since the rigidity of the concrete slab increases with an increase of its thickness.

The influence of slab thickness on the maximum tensile stress at the bottom of concrete slab (σ_i) due to circular wheel load is shown in Figure 7. From Figure 7, it can be easily observed that the tensile stress developed ate the bottom surface of the concrete slab decreases significantly with a corresponding increase of slab thickness up to 225 mm (9 inch) above which the effects of increasing slab thickness on maximum tensile stress on slab bottom diminishes. This variation of tensile stress at the bottom of concrete is easily predictable since with an increase in thickness, the inertia of the cross sectional area increases at a much higher rate than the increase in distance of bottom fiber of concrete slab from the neutral axis resulting in less tensile stress at the bottom fiber of the slab.

The influence of slab thickness on the compressive stress (σ_v) on the top of subgrade below the center of the circular loaded area is displayed in Figure 8. It is obvious from Figure 8 that with an increase in slab thickness, the corresponding vertical compressive stress on the top of subgrade is reduced and the influence is highly significant up to a thickness of 225 mm. With an increase in slab thickness, the rigidity of the slab increase resulting in less deflection of the area under the circular loaded portion of the concrete slab which results in a more uniform stress distribution on top of the base material and hence on the top of subgrade.

Effect of Thickness of Base Course

Similar to the parametric study performed for the effect of slab thickness on pavement responses as described in the preceding section, a whole series of 3D finite element analysis were performed using the reference model to observe the effect of thickness of base course material on the pavement responses. A reasonably wide range of base course thickness was considered for the parametric study, starting from 150 mm (6 inch) to 600 mm (24 inch) thickness. For different values of base thickness other geometrical properties of the reference model were kept constant. The effects are shown in (Figure 9–11).

The influence of base thickness on the vertical deflection ($\delta_{surface)}$ of the top of concrete surface below the center of the circular loaded area is shown in Figure 9. It can be easily observed from Figure 9 that the vertical deflection of the top concrete surface decreases with the increase of base thickness, but this decrease in surface deflection is very insignificant.

The influence of base thickness on the maximum tensile stress at the bottom of concrete slab (σ_i) due to circular wheel load is shown in Figure 10 and shows that the tensile stress developed at the bottom surface of the concrete slab decreases slightly with a corresponding increase of slab thickness.

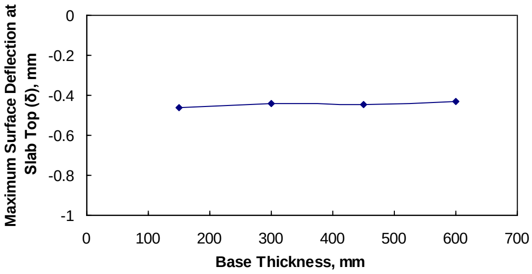

Fig. 9: *Variation of Vertical Surface Deflection of Slab Top with Base Thickness.*

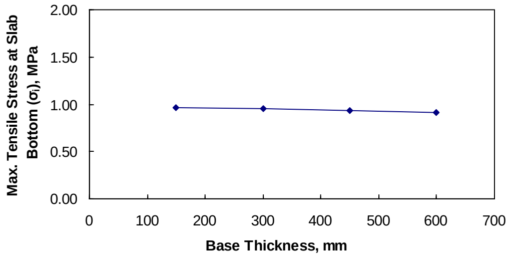

Fig. 10: *Variation of Maximum Tensile Stress at Slab Bottom with Base Thickness.*

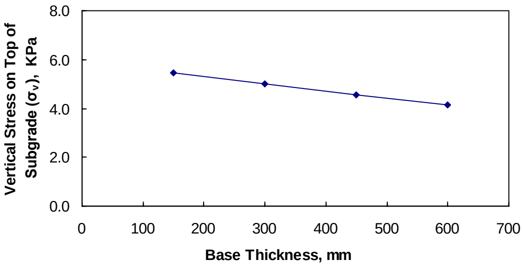

Fig. 11: *Variation of Compressive stress on Top of Subgrade with Base Thickness.*

The influence of base thickness on the compressive stress (σ_v) on the top of subgrade below the center of the circular loaded area is displayed in Figure 11. It is obvious from Figure 11 that with an increase in slab thickness, the corresponding vertical compressive stress on the top of subgrade is reduced. The influence of base thickness on the vertical stress at the top of subgrade below the centre line of the loaded area is insignificant compared to the reduction in vertical compressive stress resulting from changes in concrete slab thickness. But the increase in base thickness has more influence on compressive stress on subgrade top than the other two response i.e. vertical deflection of slab top surface and maximum tensile stress at slab bottom.

CONCLUSIONS

The effects of concrete slab thickness on critical pavement responses are highly significant up to 225 mm (9 inch). An increase in slab thickness results in a considerable decrease in vertical deflection of top surface. Maximum tensile stress developed at the bottom of the concrete slab and the vertical compressive stress on the subgrade surface also decreases significantly. Above this critical thickness of 225 mm, the influence of concrete slab thickness on critical pavement responses diminishes. Based on this finding, it is recommended to take special care in designing a concrete slab below a thickness of 225 mm.

On the contrary, effects of base course thickness on pavement responses are found to be of little importance. The increase in thickness has little effect on reducing the critical pavement stresses and deflection. So the use of a base course material is limited in extent for drainage purpose.

Nonlinear characterization of the base course material has no significant effect on the pavement responses as the stresses developed in the base course material layer is within the elastic limit. As a result, the base course material behaves as an elastic material under the present loading and geometry conditions.

Nonlinear characterization of subgrade soil has considerable effect on the pavement responses especially on the deflection of the top surface. But the relative deflection of the center of the concrete slab with respect to its edge doesn’t change significantly. As a result, the tensile stress developed at the bottom surface of the concrete slab doesn’t change significantly and is almost negligible.

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